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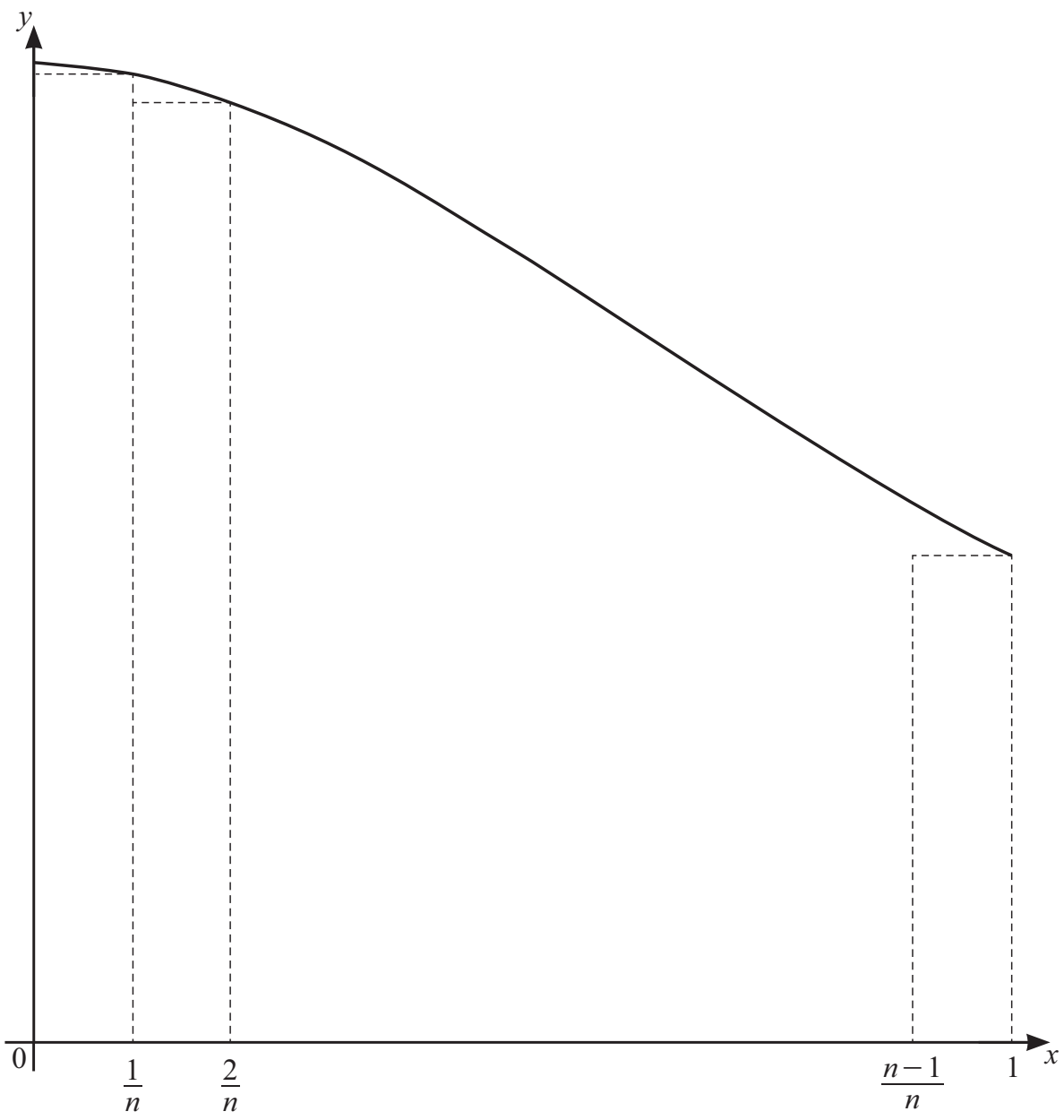

The diagram shows the curve with equation $y = \dfrac{1}{x^2+1}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\dfrac{1}{n}$.

(a) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{n}{n^2+r^2} < \frac{1}{4}\pi. \qquad [5]$$

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(b) Use a similar method to find a lower bound for $\displaystyle\sum_{r=1}^{n} \frac{n}{n^2+r^2}$. Give your answer in terms of n and π. [4]

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(c) Deduce the exact value of $\displaystyle\lim_{n\to\infty} \sum_{r=1}^{n} \frac{n}{n^2+r^2}$. [1]

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