8 **(a)** Find the values of a for which the system of equations

$$\begin{aligned} \tfrac{3}{2}x + 3y + 8z &= 1, \\ ax + 3y + 4z &= 2, \\ ay - \ z &= 3, \end{aligned}$$

does not have a unique solution. [3]

The matrix $\mathbf{A}$ is given by

$$\mathbf{A} = \begin{pmatrix} \frac{3}{2} & 3 & 8 \\ 0 & 3 & 4 \\ 0 & 0 & -1 \end{pmatrix}.$$

(b) Given that $\mathbf{B} = \mathbf{A}^{-1}$, use the characteristic equation of $\mathbf{A}$ to show that $\mathbf{B}^2 = p\mathbf{I} + q\mathbf{A}$, where p and q are constants to be determined. [4]

(c) Find a matrix $\mathbf{P}$ and a diagonal matrix $\mathbf{D}$ such that $\mathbf{A}^{-1} = \mathbf{PDP}^{-1}$. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.