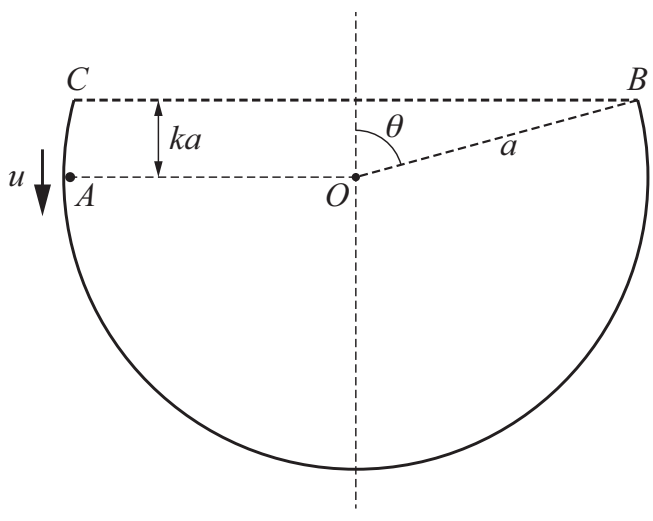
7




A fixed hollow sphere has radius $a$ and centre $O$. The points $A$, $B$ and $C$ lie on the inner surface of the sphere with $OA$ and $BC$ horizontal. A portion of the sphere has been removed by a horizontal cut through points $B$ and $C$ at a vertical distance $ka$ above the centre of the sphere, where $k$ is a positive constant and $k < 1$. The points $O$, $A$, $B$ and $C$ all lie in the same vertical plane. $OB$ makes an angle $\theta$ with the upward vertical through $O$ (see diagram).

A particle $P$ of mass $m$ is free to move on the smooth inner surface of the sphere. The particle $P$ is projected vertically downwards from $A$ with speed $u$ and begins to move in a vertical circle.

**(a)** In the case where $u = \sqrt{\frac{6}{5}ga}$, the reaction on $P$ at $B$ is half the reaction on $P$ at $A$.

Find the value of $k$. [5]

**(b)** Find an expression for $u$, in terms of $a$ and $g$, in the case that the particle just reaches $B$. [1]

**(c)** Find an expression for $u$, in terms of $a$ and $g$, in the case that the particle passes through $B$ and in its subsequent motion reaches $C$. [4]

**Additional page**

If you use the following page to complete the answer to any question, the question number must be clearly shown.