6 A bag contains 7 red balls and 3 blue balls. Kieran selects 2 balls at random, without replacement. The number of red balls selected by Kieran is denoted by X, and the number of different colours present in Kieran's selection is denoted by Y.

(a) Find the probability generating functions, $G_X(t)$ of X and $G_Y(t)$ of Y. [4]

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The random variable Z is the sum of the number of red balls and the number of different colours present in Kieran's selection. Kieran claims that the probability generating function of Z is equal to $G_X(t) \times G_Y(t)$.

(b) Explain why Kieran is incorrect. [1]

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(c) Find the probability generating function of Z, expressing your answer as a polynomial in t. [4]

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(d) Use the probability generating function of Z to find $E(Z)$. [2]

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Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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