5 **(a)** **(i)** By writing $\cos\left(\frac{1}{12}\pi\right)$ as $\cos\left(\frac{1}{4}\pi - \frac{1}{6}\pi\right)$, show that $\cos\left(\frac{1}{12}\pi\right) = \frac{1}{4}\left(\sqrt{6} + \sqrt{2}\right)$. [1]

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(ii) Show also that $\sin\left(\frac{1}{12}\pi\right) = \frac{1}{4}\left(\sqrt{6} - \sqrt{2}\right)$. [1]

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The matrix **M** is such that $\mathbf{M} = \frac{1}{4}\begin{pmatrix}\sqrt{6}+\sqrt{2} & \sqrt{2}-\sqrt{6}\\ \sqrt{6}-\sqrt{2} & \sqrt{6}+\sqrt{2}\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}$.

(b) The matrix **M** represents a sequence of two geometrical transformations in the *x*-*y* plane.

Give full details of each transformation, and make clear the order in which they are applied. [4]

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(c) Write $\mathbf{M}^{-1}$ as the product of two matrices, neither of which is **I**. [2]

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(d) Given that $y = mx$ is an invariant line of the transformation represented by **M**, show that

$$m^2 \sin\left(\tfrac{1}{12}\pi\right) + 2m\cos\left(\tfrac{1}{12}\pi\right) - \sin\left(\tfrac{1}{12}\pi\right) = 0$$

and find the values of m in the form $a\cot\left(\frac{1}{12}\pi\right) + b\,\mathrm{cosec}\left(\frac{1}{12}\pi\right)$, where a and b are integers to be determined. [6]

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