6 The matrix **P** has non-zero eigenvalues and is given by

$$\mathbf{P} = \begin{pmatrix} a & 1 & 1 \\ 0 & 2a & -1 \\ 0 & 0 & -3a \end{pmatrix}.$$

(a) State, in terms of a, the eigenvalues of **P**. [1]

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(b) **(i)** Find $\mathbf{P}^2$ in terms of a. [1]

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(ii) Use the characteristic equation of **P** to find $\mathbf{P}^{-1}$ in terms of a. [3]

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The 3×3 matrix **A** has eigenvalues 1, 2, 3 with corresponding eigenvectors

$$\begin{pmatrix} a \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2a \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ -3a \end{pmatrix},$$

respectively.

(c) Find **A** in terms of a. [5]

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