8 **(a)** State the sum of the series $z + z^2 + ... + z^n$, for $z \neq 1$. [1]

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(b) By letting $z = \frac{1}{2}(\cos\theta + i\sin\theta)$ use de Moivre's theorem to deduce that

$$\sum_{m=1}^{n}\left(\frac{1}{2}\right)^m \sin m\theta = \frac{\left(\frac{1}{2}\right)^{n+2}\sin n\theta - \left(\frac{1}{2}\right)^{n+1}\sin(n+1)\theta + \frac{1}{2}\sin\theta}{\frac{5}{4} - \cos\theta}.$$ [6]

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(c) Use the result in **(b)** to find $\sum_{m=1}^{n}\left(\frac{1}{2}\right)^m m\cos m\theta$ in terms of n and θ. [You do not need to simplify your answer.] [3]

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(d) Hence find $\sum_{m=1}^{\infty}\left(\frac{1}{2}\right)^m m\cos m\theta$ in terms of $\cos\theta$. [You may assume that $\frac{n}{2^n} \to 0$ as $n \to \infty$.] [2]

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Additional page

If you use the following lined page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.

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