4 **(a)** By considering the binomial expansion of $\left(z+\frac{1}{z}\right)$, where $z=\cos\theta+\mathrm{i}\sin\theta$, use de Moivre's theorem to show that

$$\cos^6\theta = a\cos 6\theta + b\cos 4\theta + c\cos 2\theta + d,$$

where a, b, c and d are constants to be determined. [5]

(b) Find the exact value of $\int_0^{\frac{1}{2}\pi} \cos^6 2x \,\mathrm{d}x$. [3]