6 **(a)** Use the substitution $x = \frac{1}{2}\sqrt{2}\sinh u$ to find $\int \frac{1}{\sqrt{2x^2+1}}\,\mathrm{d}x$. [3]

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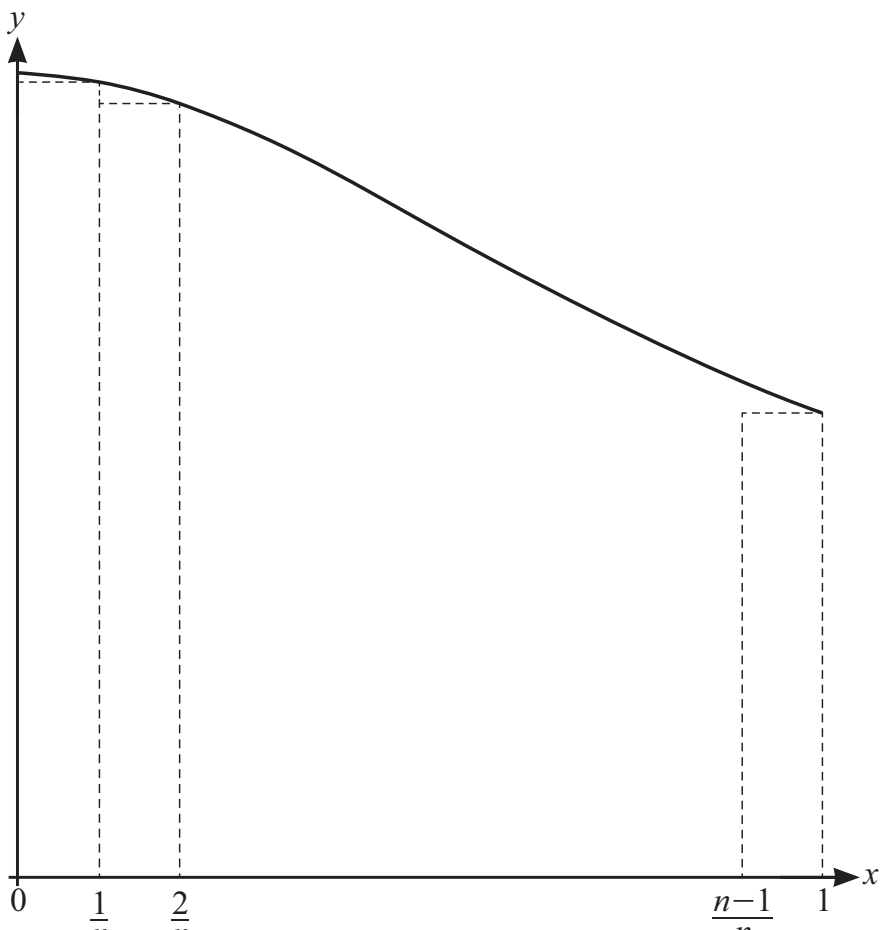

The diagram shows the curve with equation $y = \dfrac{1}{\sqrt{2x^2+1}}$ for $0 \leqslant x \leqslant 1$, together with a set of n rectangles of width $\dfrac{1}{n}$.

(b) By considering the sum of the areas of these rectangles, show that

$$\sum_{r=1}^{n} \frac{1}{\sqrt{2r^2+n^2}} < \tfrac{1}{2}\sqrt{2}\ln\left(\sqrt{2}+\sqrt{3}\right). \qquad [5]$$

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(c) Use a similar method to find, in terms of n, a lower bound for $\sum_{r=1}^{n} \frac{1}{\sqrt{2r^2+n^2}}$. [4]

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(d) Deduce the exact value of $\lim_{n\to\infty} \sum_{r=1}^{n} \frac{1}{\sqrt{2r^2+n^2}}$. [1]

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