8 **(a)** Find the values of k for which the system of equations

$$\begin{aligned} x-y+2kz &= 1, \\ kx+y+\ 2z &= 2, \\ 2x-y+\quad z &= 3, \end{aligned}$$

does not have a unique solution. [3]

(b) Given that $k=-\frac{1}{2}$, show that the system of equations in part **(a)** is inconsistent. Interpret this situation geometrically. [3]

(c) Given instead that $k=-1$, show that the system of equations in part **(a)** is also inconsistent. Interpret this situation geometrically. [4]

The matrix **A** is given by

$$\mathbf{A}=\begin{pmatrix} 1 & -1 & -2 \\ -1 & 1 & 2 \\ 2 & -1 & 1 \end{pmatrix}.$$

(d) Use the characteristic equation of **A** to show that $\mathbf{A}^4 = p\mathbf{A}^2 + q\mathbf{A}$ where p and q are integers to be determined. [5]

Additional page

If you use the following page to complete the answer(s) to any question(s), the question number(s) must be clearly shown.