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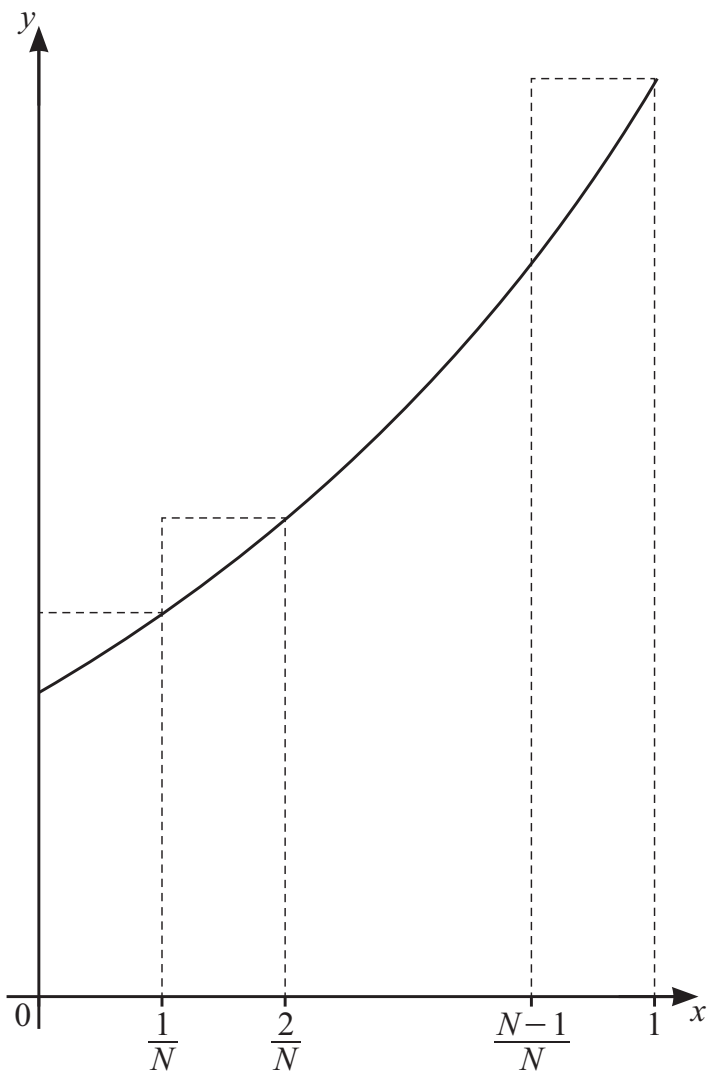

The diagram shows the curve with equation $y = \frac{1}{2}\left(3^x\right)$ for $0 \leqslant x \leqslant 1$, together with a set of N rectangles each of width $\frac{1}{N}$.

(a) By considering the sum of the areas of these rectangles, show that $\int_0^1 \frac{1}{2}\left(3^x\right)\mathrm{d}x < U_N$, where

$$U_N = \frac{3^{\frac{1}{N}}}{N\left(3^{\frac{1}{N}} - 1\right)}. \qquad [4]$$

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(b) Use a similar method to find, in terms of N, a lower bound L_N for $\int_0^1 \frac{1}{2}\left(3^x\right)\mathrm{d}x$. [4]

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(c) By simplifying $U_N - L_N$, show that $\lim_{N\to\infty}\left(U_N - L_N\right) = 0$. [2]

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