7 The curve C has parametric equations

$$x = 8\ln(\tan\tfrac{1}{2}t) - 3\cot t - 3t, \qquad y = 6\ln(\tan\tfrac{1}{2}t) + 4\cot t + 4t, \qquad \text{for } \tfrac{1}{6}\pi \leqslant t \leqslant \tfrac{1}{3}\pi.$$

(a) **(i)** Show that $\dfrac{dx}{dt} = 8\operatorname{cosec} t + 3\cot^2 t$. [1]

(ii) Find $\dfrac{dy}{dt}$. [1]

(iii) Find the exact value of the length of C. [6]

(b) Show that

$$\frac{d^2y}{dx^2} = \frac{a\cot t\operatorname{cosec} t(\operatorname{cosec}^2 t + 1)}{(b\operatorname{cosec} t + c\cot^2 t)^n},$$

where a, b, c and n are integers to be determined. [5]