8 **(a)** Find the set of values of a for which the system of equations

$$\begin{aligned} 5x + ay \quad &= 3, \\ 20x - 5y \quad &= 2, \\ 2x - 3y + z &= 1, \end{aligned}$$

has a unique solution and interpret this situation geometrically. [3]

(b) Given that $a = -\frac{5}{4}$, show that the system of equations in part **(a)** is inconsistent and interpret this situation geometrically. [3]

The matrix **A** is given by

$$\mathbf{A} = \begin{pmatrix} 5 & 0 & 0 \\ 20 & -5 & 0 \\ 2 & -3 & 1 \end{pmatrix}.$$

(c) Find a matrix **P** and a diagonal matrix **D** such that $(\mathbf{A} + 3\mathbf{I})^2 = \mathbf{PDP}^{-1}$. [7]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.