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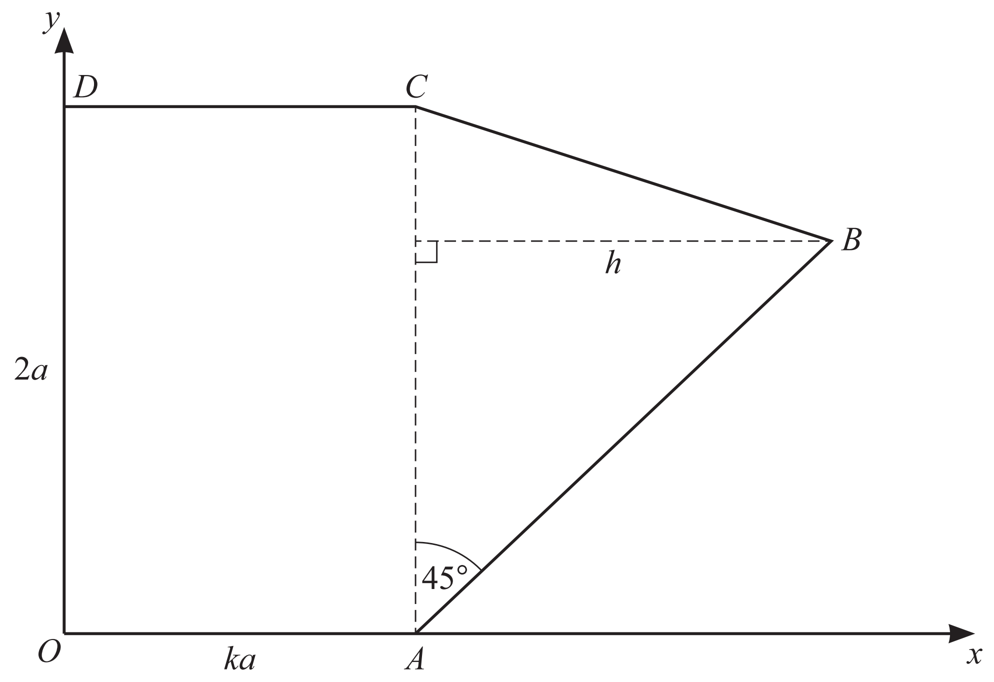

A uniform lamina *OABCD* consists of a rectangle *OACD* and a triangle *ABC*. The length of *OA* is *ka*, the length of *OD* is $2a$, the height of triangle *ABC* is *h* and angle *CAB* is 45° (see diagram). Relative to axes through *O*, parallel and perpendicular to *OA* as shown, the centre of mass of triangle *ABC* is $(\overline{x}, \overline{y})$.

(a) Show that $\overline{x}$ is $\frac{1}{3}(3ka+h)$, and find an expression for $\overline{y}$. [3]

The lamina *OABCD* is placed vertically on its edge *OA* on a horizontal plane.

(b) Find, in terms of *a* and *k*, the set of values of *h* for which the lamina is in equilibrium. [4]

It is now given that $k = \frac{\sqrt{3}}{3}$ and that the lamina is on the point of toppling.

(c) Find, in terms of *a*, the coordinates of the centre of mass of the triangle *ABC*. [2]