7 A discrete random variable X takes values $r = 0, 1, 2$ with probabilities $\mathrm{P}(X = r)$ as given in the following table.

r	0	1	2
$\mathrm{P}(X = r)$	a	$2a$	b

(a) Write down the probability generating function of X, and use it to find an expression for $\mathrm{E}(X)$ in terms of a and b. [2]

(b) Show that $\mathrm{Var}(X) = 2b + 2(a + b)(1 - 2a - 2b)$. [3]

The random variable Y is defined by $Y = X_1 + X_2 + X_3 + \cdots + X_{10}$ where $X_1, X_2, X_3, \ldots, X_{10}$ are ten independent observations of X.

(c) Using the probability generating function of Y, and your answer to part **(a)**, show that $\mathrm{E}(Y) = 10\mathrm{E}(X)$. [3]

(d) For the case $b = 0$, define fully the distribution of Y. [2]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.