4 The random variable X takes values 1 and 2 with probabilities $\frac{2}{5}$ and $\frac{3}{5}$ respectively.

(a) Write down the probability generating function $G_X(t)$ of X. [1]

The random variable Y is the sum of four independent observations of X.

(b) Find the probability generating function $G_Y(t)$ of Y. Give your answer in the form $G_Y(t) = at^m(b+ct)^n$, where a, b, c, m and n are constants to be determined. [2]

(c) Use $G_Y(t)$ to find $P(Y = 6)$. [2]

(d) Find $\mathrm{Var}(Y)$. [5]