6 **(a)** Given that $\int_{-2a}\left(\frac{1}{2}e^{2x}+\frac{1}{4}e^{-x}\right)dx = 5$, where a is a positive constant, show that

$$a = \tfrac{1}{2}\ln\left(10+\tfrac{1}{2}e^{-a}+\tfrac{1}{2}e^{-4a}\right).$$ [4]

(b) Hence show by calculation that the value of a lies between 1.0 and 1.2. [2]

(c) Use the iterative formula

$$a_{n+1} = \tfrac{1}{2}\ln\left(10+\tfrac{1}{2}e^{-a_n}+\tfrac{1}{2}e^{-4a_n}\right)$$

to find the value of a correct to 4 significant figures. Give the result of each iteration to 6 significant figures. [3]