

6. The function f has derivatives of all orders for all real numbers. It is known that $f(0) = 2$, $f'(0) = 3$,

$$f''(x) = -f(x^2), \text{ and } f'''(x) = -2x \cdot f'(x^2).$$

(a) Find $f^{(4)}(x)$, the fourth derivative of f with respect to x . Write the fourth-degree Taylor polynomial for f about $x = 0$. Show the work that leads to your answer.

(b) The fourth-degree Taylor polynomial for f about $x = 0$ is used to approximate $f(0.1)$. Given that

$$\left| f^{(5)}(x) \right| \leq 15 \text{ for } 0 \leq x \leq 0.5, \text{ use the Lagrange error bound to show that this approximation is within } \frac{1}{10^5} \text{ of the exact value of } f(0.1).$$

(c) Let g be the function such that $g(0) = 4$ and $g'(x) = e^x f(x)$. Write the second-degree Taylor polynomial for g about $x = 0$.

Write your responses to this question only on the designated pages in the separate Free Response booklet. Write your solution to each part in the space provided for that part.

STOP

END OF EXAM