



Note: Figure not drawn to scale.

2. A block of mass  $m$  starts at rest at the top of an inclined plane of height  $h$ , as shown in the figure above. The block travels down the inclined plane and makes a smooth transition onto a horizontal surface. While traveling on the horizontal surface, the block collides with and attaches to an ideal spring of spring constant  $k$ . There is negligible friction between the block and both the inclined plane and the horizontal surface, and the spring has negligible mass. Express all algebraic answers for parts (a), (b), and (c) in terms of  $m$ ,  $h$ ,  $k$ , and physical constants, as appropriate.

(a)

- i. Derive an expression for the speed of the block just before it collides with the spring.
- ii. Is the speed halfway down the incline greater than, less than, or equal to one-half the speed at the bottom of the inclined plane?

\_\_\_\_\_ Greater than      \_\_\_\_\_ Less than      \_\_\_\_\_ Equal to

Justify your answer.

(b) Derive an expression for the maximum compression of the spring.

(c) Determine an expression for the time from when the block collides with the spring to when the spring reaches its maximum compression.

The block is again released from rest at the top of the incline, and when it reaches the horizontal surface it is moving with speed  $v_0$ . Now suppose the block experiences a resistive force as it slides on the horizontal surface.

The magnitude of the resistive force  $F$  is given as a function of speed  $v$  by  $F = \beta v^2$ , where  $\beta$  is a positive constant with units of  $\text{kg/m}$ .

(d)

- i. Write, but do NOT solve, a differential equation for the speed of the block on the horizontal surface as a function of time  $t$  before it reaches the spring. Express your answer in terms of  $m$ ,  $h$ ,  $k$ ,  $\beta$ ,  $v$ , and physical constants, as appropriate.

- ii. Using the differential equation from part (d)i, show that the speed of the block  $v(t)$  as a function of time  $t$  can be written in the form  $\frac{1}{v(t)} = \frac{1}{v_0} + \frac{\beta t}{m}$ , where  $v_0$  is the speed at  $t = 0$ .

(e) Sketch graphs of position  $x$  as a function of time  $t$ , velocity  $v$  as a function of time  $t$ , and acceleration  $a$  as a function of time  $t$  for the block as it is moving on the horizontal surface before it reaches the spring.

