

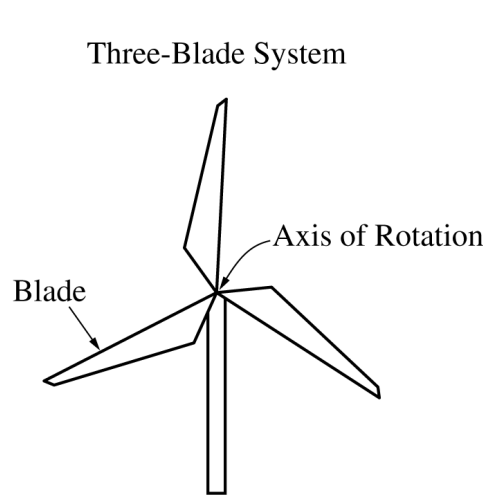


Figure 1

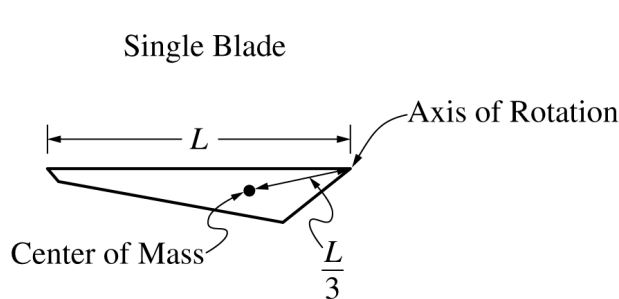


Figure 2

A wind turbine includes a three-blade system that rotates about an axis through the end of each blade, as shown in Figure 1. Each blade has a length L and mass M , with a center of mass located at a distance $\frac{L}{3}$ from the axis of rotation, as shown in Figure 2.

- (a) Derive an expression for the rotational inertia of the three-blade system. Express your answer in terms of M , L , and physical constants, as appropriate. The rotational inertia of each blade about an axis through its center of mass is given by the equation $I_{\text{cm}} = \frac{1}{18}ML^2$.

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Continue your response to **QUESTION 3** on this page.

- (b) While the wind blows, the three-blade system operates at a constant angular speed $\omega_0 = 2.6$ rad/s. The length of one blade is $L = 36$ m. The numerical value of the rotational inertia of the system is $I_{\text{sys}} = 6.7 \times 10^6$ kg·m². Calculate the time T it takes the outer edge of a single blade to complete one revolution.

- (c) When the wind stops blowing, the angular speed of the system decreases. The angular speed ω of the system while slowing down is given as a function of time t by the equation $\omega = \omega_0 e^{-\beta_0 t}$, where β_0 is a constant with appropriate units, as shown on the graph in Figure 3.

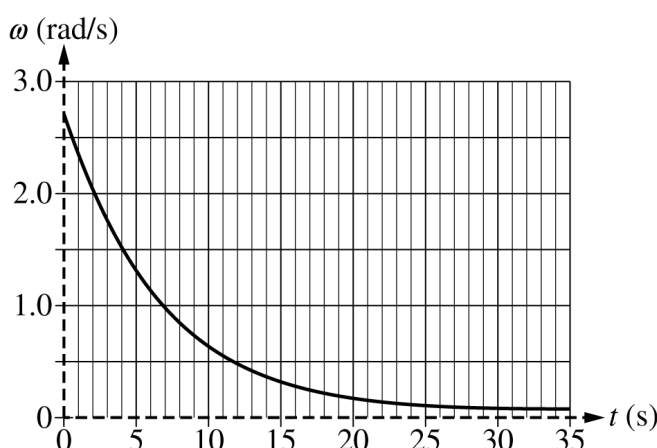


Figure 3

- i. Calculate the amount of energy dissipated from $t = 0$, when the wind stops blowing, until the system comes to rest.

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- ii. Derive an expression for the net torque exerted on the system as a function of t as the system slows down. Express your answers in terms of β_0 , ω_0 , M , L , I_{sys} , and physical constants, as appropriate.

- iii. Derive an expression for the angular displacement of the system $\Delta\theta$ as a function of t . Express your answer in terms of β_0 , ω_0 , M , L , I_{sys} , and physical constants, as appropriate.

The three-blade system is now replaced with a second three-blade system identical to the first, except that the second three-blade system slows down according to the equation $\omega = \omega_0 e^{-\beta t}$, where $\omega_0 = 2.6$ rad/s and $\beta > \beta_0$. The original angular speed function is shown as a dashed line in Figure 4.

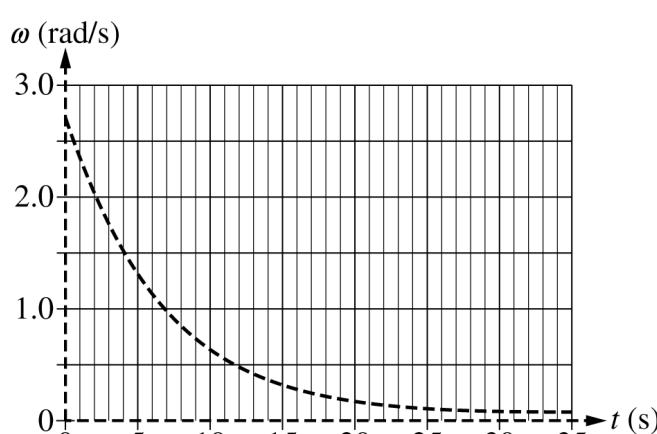


Figure 4

- (d) On the graph in Figure 4, sketch the angular speed of the second three-blade system as a function of time t .

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