2.1 Hyperbolic functions

Name: ____________ Class: ________ Date: __________ **Total: 13 marks**

Objective

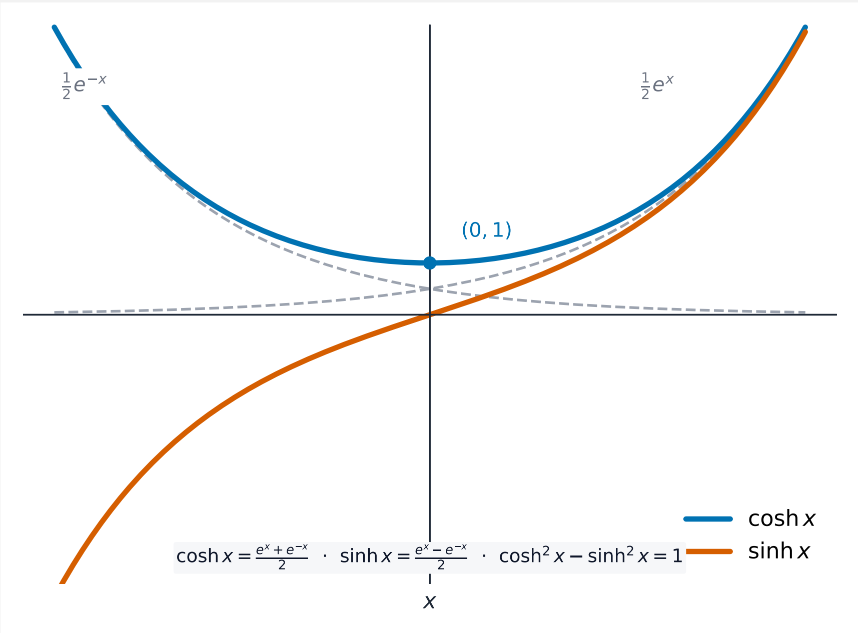
Build the skills to answer exam questions on **hyperbolic functions** 双曲函数—the definitions, identities, and the **logarithmic form** 对数形式 of the inverses.

You must be able to:

- use $\cosh x = \frac{1}{2}(e^x + e^{-x})$, $\sinh x = \frac{1}{2}(e^x - e^{-x})$
- use $\cosh^2 x - \sinh^2 x = 1$ and related identities
- use $\sinh^{-1} x = \ln(x + \sqrt{x^2+1})$ and solve equations

1 Worked examples

Identity & solving



cosh *is the average of* e^x *and* e^{-x}; sinh *is half their difference*

$$\cosh^2 x - \sinh^2 x = \frac{(e^x+e^{-x})^2 - (e^x-e^{-x})^2}{4} = \frac{4}{4} = 1.$$

Solve $\cosh x = 2$**:** $\dfrac{e^x+e^{-x}}{2} = 2 \Rightarrow e^{2x} - 4e^x + 1 = 0 \Rightarrow e^x = 2 \pm \sqrt{3}$, $x = \ln(2 \pm \sqrt{3})$.

2 Practice

2.1 Write down the value of $\cosh 0$. [1]

2.2 Evaluate $\sinh^{-1} 0$ using the logarithmic form. [1]

3 Exam-style questions

3.1 The identity satisfied by hyperbolic functions is: [1]

- **A** $\cosh^2 x + \sinh^2 x = 1$
- **B** $\cosh^2 x - \sinh^2 x = 1$
- **C** $\sinh^2 x - \cosh^2 x = 1$
- **D** $\cosh^2 x - \sinh^2 x = -1$

3.2 Solve the equation $\cosh x = 3$, giving exact answers. [4]

3.3 (a) Show that $\sinh^{-1} x = \ln(x + \sqrt{x^2 + 1})$. [4]

(b) Hence evaluate $\sinh^{-1}(\frac{3}{4})$ as a single logarithm. [2]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **2.1 Hyperbolic functions** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/21 N25 —Q7 (hyperbolic functions)
- 9231/24 N25 —Q2 (hyperbolic / inverse)
- 9231/21 J25 —Q6 (hyperbolic functions)

Solutions

2.1 $\cosh 0 = \frac{1}{2}(1+1) = 1$.

2.2 $\sinh^{-1} 0 = \ln(0 + \sqrt{0+1}) = \ln 1 = 0$.

3.1 **B**. $\cosh^2 x - \sinh^2 x = 1$.

3.2 $\dfrac{e^x + e^{-x}}{2} = 3 \Rightarrow e^{2x} - 6e^x + 1 = 0$; $e^x = \dfrac{6 \pm \sqrt{32}}{2} = 3 \pm 2\sqrt{2}$; $x = \ln(3 \pm 2\sqrt{2})$.

3.3 (a) let $y = \sinh^{-1} x$, so $x = \sinh y = \frac{1}{2}(e^y - e^{-y})$; $e^{2y} - 2xe^y - 1 = 0 \Rightarrow e^y = x + \sqrt{x^2+1}$ (positive root); $y = \ln(x + \sqrt{x^2+1})$.
(b) $\sinh^{-1} \frac{3}{4} = \ln\left(\frac{3}{4} + \sqrt{\frac{9}{16} + 1}\right) = \ln\left(\frac{3}{4} + \frac{5}{4}\right) = \ln 2$.