

2.3 Differentiation (Maclaurin, hyperbolic, parametric)

Name: _____ Class: _____ Date: _____

Total: 14 marks

Objective

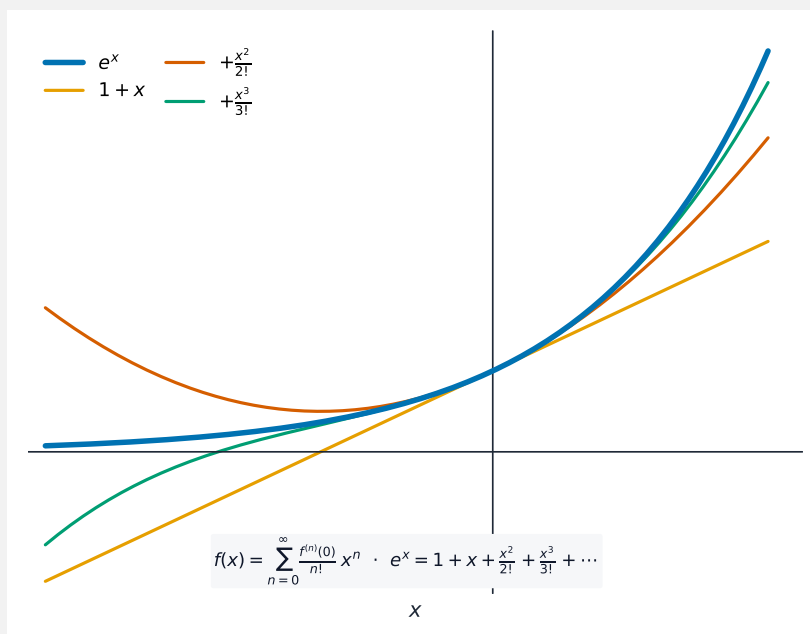
Build the skills to answer exam questions on **further differentiation** 微分—derivatives of inverse and **hyperbolic functions** 双曲函数, second derivatives of implicit/parametric curves, and **Maclaurin's series** 麦克劳林级数.

You must be able to:

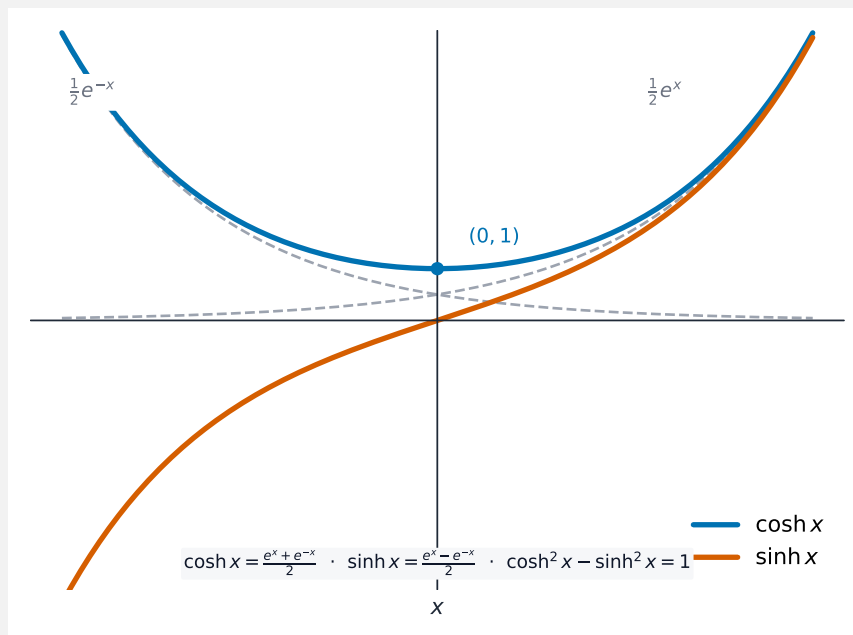
- differentiate $\sin^{-1} x$, $\tan^{-1} x$, $\sinh x$, $\cosh x$, $\tanh x$
- find $\frac{d^2y}{dx^2}$ for implicit/parametric curves
- find a Maclaurin series for a function

1 Worked examples

■ Maclaurin series



Adding more terms makes the polynomial match the function over a wider stretch around $x = 0$



The hyperbolic functions whose derivatives appear here

Maclaurin: $f(x) = f(0) + f'(0)x + \frac{f''(0)}{2!}x^2 + \dots$; e.g. $e^x = 1 + x + \frac{x^2}{2!} + \dots$.

Derivatives: $\frac{d}{dx} \sinh x = \cosh x$; $\frac{d}{dx} \tanh^{-1} x = \frac{1}{1 + x^2}$.

2 Practice

2.1 Differentiate $y = \cosh(3x)$. [2]

2.2 Differentiate $y = \sin^{-1}(2x)$. [2]

3 Exam-style questions

3.1 $\frac{d}{dx} \tanh x$ equals: [1]

- **A** $\operatorname{sech}^2 x$
- **B** $\operatorname{cosech}^2 x$
- **C** $\cosh^2 x$

- $\mathbf{D} \sinh^2 x$

3.2 Find the Maclaurin series for $\ln(1+x)$ up to and including the term in x^3 . [4]

3.3 A curve is given by $x = t^2$, $y = t^3$. Find $\frac{d^2y}{dx^2}$ in terms of t . [5]

4 Go further

You are now ready for the real exam questions on this subtopic. Open the **2.3 Differentiation** past-paper sheet in the Library, or try these in **Practice mode**:

- 9231/21 N25 —Q2 (Maclaurin / differentiation)
- 9231/22 N25 —Q3 (differentiation)
- 9231/22 N25 —Q7 (parametric second derivative)

Solutions

2.1 $\frac{dy}{dx} = 3 \sinh(3x).$

2.2 $\frac{dy}{dx} = \frac{2}{\sqrt{1-4x^2}}.$

3.1 A. $\frac{d}{dx} \tanh x = \operatorname{sech}^2 x.$

3.2 $f(0) = 0; f'(x) = \frac{1}{1+x}, f'(0) = 1; f''(x) = -\frac{1}{(1+x)^2}, f''(0) = -1; f'''(0) = 2;$

$\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots.$

3.3 $\frac{dx}{dt} = 2t, \frac{dy}{dt} = 3t^2, \text{ so } \frac{dy}{dx} = \frac{3t^2}{2t} = \frac{3t}{2}; \frac{d^2y}{dx^2} = \frac{d}{dt} \left(\frac{3t}{2} \right) \div \frac{dx}{dt} = \frac{3/2}{2t} = \frac{3}{4t}.$