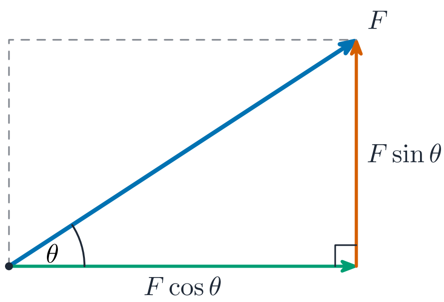
Mechanics

A-Level Mathematics

This handout covers Topic 4: **Mechanics 力学**. It studies how forces make objects move. Throughout this topic, take the acceleration of free fall as $g = 10$ m s^{-2}.

Forces and equilibrium

A **force 力** is a push or a pull. It is a vector, so it has size and direction. Because it is a vector, you can split a force into **components 分量** (usually horizontal and vertical), and you can add several forces into one **resultant 合力**.



A force at angle θ has a horizontal part $F\cos\theta$ and a vertical part $F\sin\theta$.

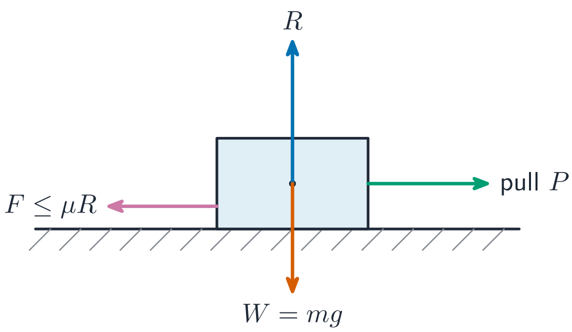
A particle is in **equilibrium 平衡** when the forces are balanced: the vector sum of the forces is zero. In practice this means the components in any direction add to zero.

Friction

When two surfaces touch, the **contact force 接触力** between them has two parts: the **normal reaction 法向反作用力** R, at right angles to the surface, and the **friction 摩擦力** F, along the surface, which opposes sliding. A "smooth" surface is a model with no friction.

Friction can only grow up to a maximum. At that maximum the body is about to slip —this is **limiting friction 最大静摩擦力**. The maximum is set by the **coefficient of friction 摩擦系数** μ:

$$F \leqslant \mu R, \qquad \text{with } F = \mu R \text{ at the point of slipping.}$$



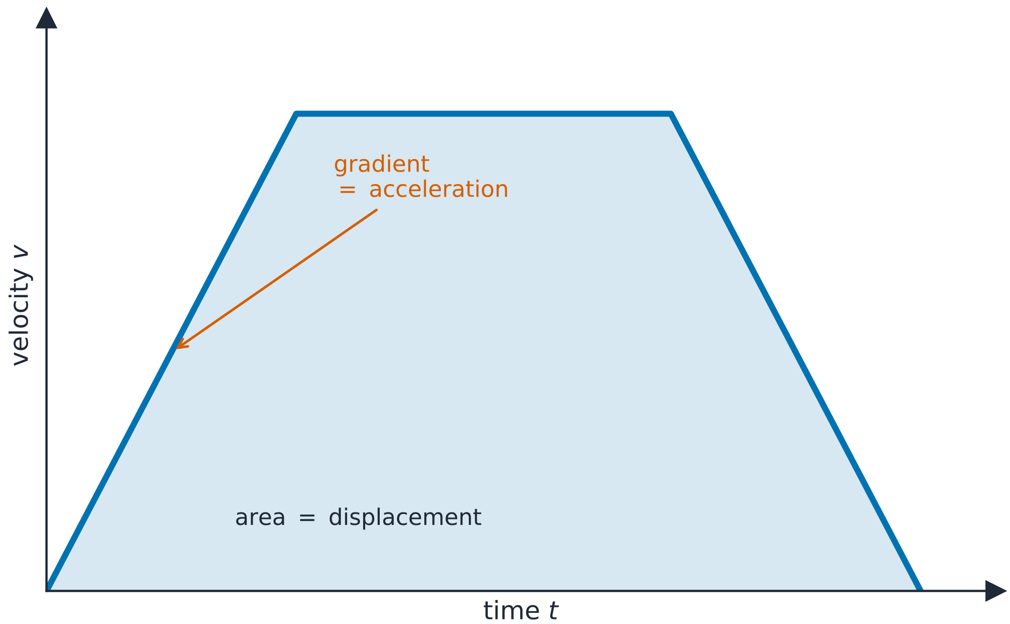
On a rough surface the contact force splits into the normal reaction R and friction F (at most μR).

By **Newton's third law** 牛顿第三定律, the two surfaces push on each other with equal and opposite forces.

Kinematics of motion in a straight line

Distance 距离 and **speed** 速率 are scalars (size only). **Displacement** 位移, **velocity** 速度 and **acceleration** 加速度 are vectors (size and direction).

On a **velocity-time graph** 速度时间图, the area under the graph is the displacement and the gradient is the acceleration. On a displacement-time graph, the gradient is the velocity. More generally, differentiate with respect to time to go from displacement to velocity to acceleration, and integrate to go back.



The shaded area gives the distance travelled; the slope of the line gives the acceleration.

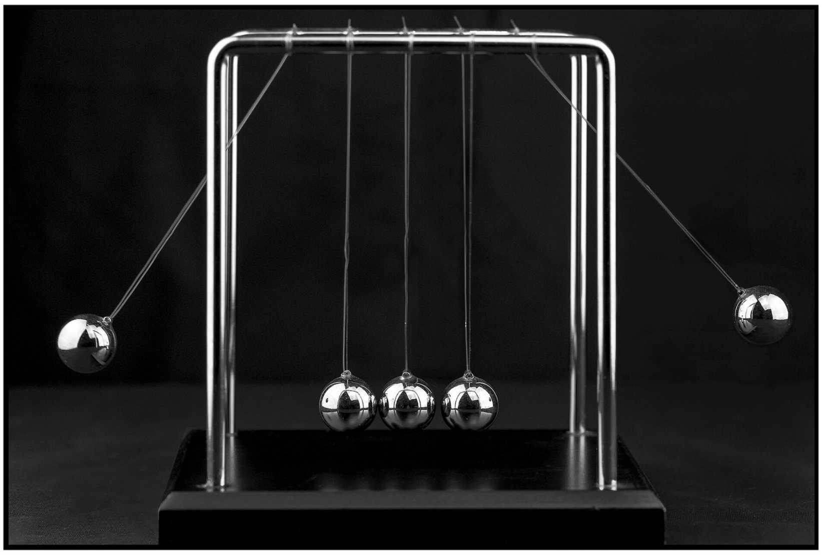
For motion with **constant acceleration** 匀加速, use these formulae (the "suvat" equations):

$$v = u + at, \qquad s = ut + \tfrac{1}{2}at^2, \qquad v^2 = u^2 + 2as, \qquad s = \tfrac{1}{2}(u+v)t.$$

Worked example. A car starts from rest and accelerates at 2.5 m s^{-2} for 4 s. Find its speed and the distance travelled.

$$v = 0 + 2.5 \times 4 = 10 \text{ m s}^{-1}, \qquad s = 0 + \tfrac{1}{2}(2.5)(4^2) = 20 \text{ m}.$$

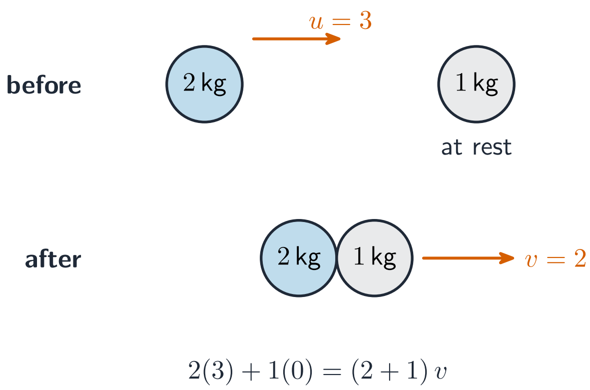
Momentum



A Newton's cradle demonstrates conservation of momentum in collisions.

The **momentum 动量** of a body is mass × velocity. It is a vector. In a direct collision of two bodies, the total momentum is unchanged. This is the **conservation of momentum 动量守恒**:

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2.$$



The total momentum is the same before and after the collision.

Worked example. A body of mass 2 kg moving at 3 m s^{-1} hits a stationary body of mass 1 kg, and they stick together. Find their common speed afterwards.

$$2(3) + 1(0) = (2+1)v \;\Rightarrow\; v = \frac{6}{3} = 2 \text{ m s}^{-1}.$$

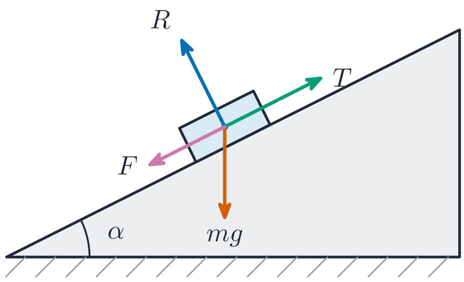
Newton's laws of motion

Newton's laws of motion 牛顿运动定律 connect force and acceleration. The key one is: resultant force = **mass** 质量 × acceleration,

$$F = ma.$$

The **weight** 重力 of a body is the force of gravity on it: $W = mg$. Forces in a problem may include weight, friction, **tension** 张力 in a string, and the push in a rod.

For motion on an **inclined plane** 斜面, split each force into a part along the slope and a part at right angles to it, then apply $F = ma$ along the slope. For **connected particles** 连接质点 (joined by a string), apply $F = ma$ to each body, or to the whole system.



On a slope, resolve the forces along the slope and at right angles to it.

Worked example. A block of mass 12 kg is pulled up a rough plane by a rope parallel to the slope. The plane is at 20° to the horizontal, the coefficient of friction is 0.4, and the acceleration is 2 m s^{-2}. Find the tension in the rope.

The normal reaction is $R = mg\cos 20° = 120\cos 20° = 112.8$ N, so the friction is $F = \mu R = 0.4 \times 112.8 = 45.1$ N. Along the slope, $T - mg\sin 20° - F = ma$:

$$T = ma + mg\sin 20° + F = 12(2) + 120\sin 20° + 45.1 = 24 + 41.0 + 45.1 = 110 \text{ N (3 s.f.)}.$$

Energy, work and power

A roller coaster trades potential energy for kinetic energy as it rises and falls.

The **work done** 功 by a constant force is the force times the distance moved in the direction of the force: $W = Fd\cos\theta$, where θ is the angle between the force and the motion. Work is measured in joules (J).

Energy comes in forms you can calculate:

- **Kinetic energy** 动能 (energy of movement): $\mathrm{KE} = \frac{1}{2}mv^2$.
- **Gravitational potential energy** 重力势能 (energy of height): $\mathrm{PE} = mgh$.

The work done by the outside forces equals the change in the total energy. When no friction acts, the total energy stays the same —the **conservation of energy** 能量守恒.

Power 功率 is the rate of doing work. For a force pulling in the direction of motion, $P = Fv$ (power = force × velocity). Power is measured in watts (W).

Worked example. A car engine works at 12 kW while the car moves at 20 m s^{-1} on a level road. Find the driving force.

$$P = Fv \;\Rightarrow\; F = \frac{P}{v} = \frac{12000}{20} = 600\ \mathrm{N}.$$