Physical quantities and units

A-Level Physics

Physical quantities

A **physical quantity** 物理量 has two parts: a number (its **magnitude** 大小) and a **unit** 单位. The number on its own tells you nothing. You must also say what is measured and in which unit.

Example: "the length is 1.5" is not complete. "The length is 1.5 m" is a physical quantity.

Making estimates

You should be able to **estimate** 估算 the size of the physical quantities in this syllabus. Learn these rough values:

- **mass** 质量 of an adult human: ~ 70 kg
- **weight** 重力 of an adult human: ~ 700 N
- height of an adult human: ~ 1.7 m
- mass of an apple: ~ 0.1 kg (so its weight is about 1 N)
- speed of sound in air: ~ 340 m s^{-1}
- speed of light in a **vacuum** 真空: 3.0×10^8 m s^{-1}
- **acceleration** 加速度 of **free fall** 自由落体: $g \approx 9.81$ m s^{-2}

A good estimate has the right **order of magnitude** 数量级 (the right power of ten). For a human, 70 kg is a good guess; 7 kg is not.

SI units 国际单位制

Base units

There are five **base quantities** 基本量 in the SI system. You must know them and their units:

- **mass** —kilogram, kg
- **length** 长度—metre, m
- time —second, s
- **current** 电流—ampere 安培, A
- **temperature** 温度—kelvin 开尔文, K

Every other unit in this syllabus is built from these five.

Derived units

A **derived unit** 导出单位 is made by multiplying or dividing base units. You should be able to write any quantity in this syllabus in base units.

Build a derived unit from the equation that defines it:

- **speed** 速率 = distance / time, so its unit is m s^{-1}
- **acceleration** = change in **velocity** 速度 / time, so its unit is m s^{-2}
- **force** 力 = mass × acceleration, so its unit is kg m s^{-2}. The **newton** 牛顿 is 1 N = 1 kg m s^{-2}.
- **work** 功 and **energy** 能量 = force × distance, so the unit is kg m^2 s^{-2}. The **joule** 焦耳 is 1 J = 1 kg m^2 s^{-2}.
- **power** 功率 = energy / time, so the unit is kg m^2 s^{-3}. The **watt** 瓦特 is 1 W = 1 kg m^2 s^{-3}.
- **pressure** 压强 and **stress** 应力 = force / area, so the unit is kg m^{-1} s^{-2}. The **pascal** 帕斯卡 is 1 Pa = 1 kg m^{-1} s^{-2}.

When a question asks for the SI base units of a quantity, replace each named unit with its base units, then simplify. Example: the SI base units of the watt are kg m^2 s^{-3}.

Checking that the units match

An equation is **homogeneous** 量纲一致 when both sides have the same base units. In plain words: the units on both sides match.

Write each side in base units and compare. Take the equation $v^2 = u^2 + 2as$:

- left side: $(\text{m s}^{-1})^2 = \text{m}^2\ \text{s}^{-2}$
- right side, first term: $(\text{m s}^{-1})^2 = \text{m}^2\ \text{s}^{-2}$
- right side, second term: $\text{m s}^{-2} \cdot \text{m} = \text{m}^2\ \text{s}^{-2}$

Both sides give m^2 s^{-2}, so the units match.

Be careful: matching units do **not** prove the whole equation is correct. It could still have a wrong number, or a missing factor of 2. But if the units do **not** match, the equation is wrong for sure.

Prefixes

A **prefix** 词头 is a letter put in front of a unit to make it bigger or smaller by powers of ten. You must know these:

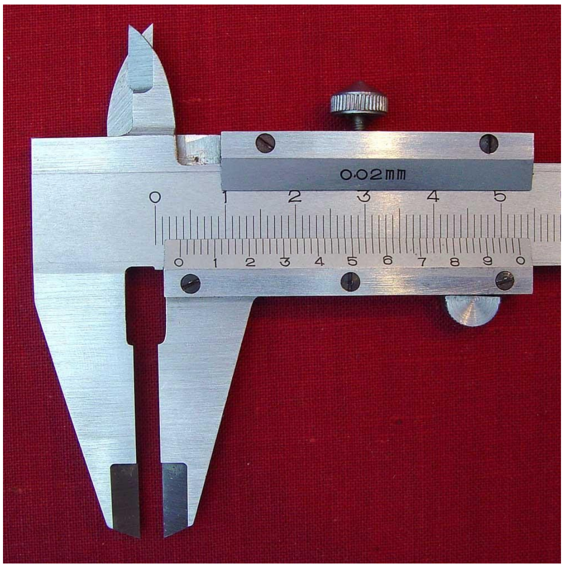
Prefix	**Symbol**	**Factor**
tera	T	10^{12}
giga	G	10^{9}
mega	M	10^{6}
kilo	k	10^{3}
deci	d	10^{-1}
centi	c	10^{-2}
milli	m	10^{-3}
micro	μ	10^{-6}
nano	n	10^{-9}
pico	p	10^{-12}

To change a prefixed unit into base units, replace the prefix with its factor, then simplify. Example: change 0.25 kN mm^{-2} into N m^{-2}:

$$0.25 \text{ kN mm}^{-2} = 0.25 \times \frac{10^3 \text{ N}}{(10^{-3} \text{ m})^2} = 0.25 \times \frac{10^3}{10^{-6}} \text{ N m}^{-2} = 2.5 \times 10^8 \text{ N m}^{-2}.$$

Take special care with squared units like mm^2: you must square the factor too.

Errors and uncertainties



A vernier caliper measures length precisely, with a small uncertainty.

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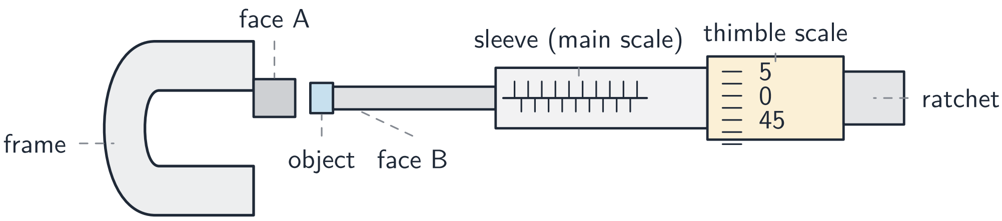
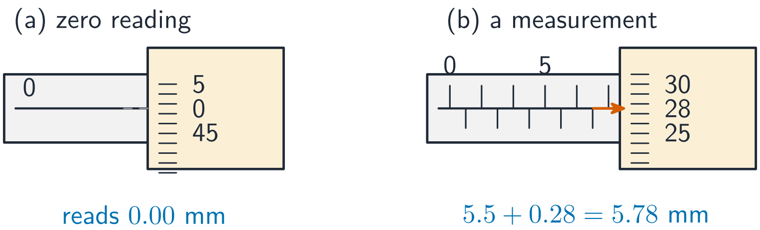
Every **measurement** 测量 has some **uncertainty** 不确定度—we are never fully sure of the value. A good experimenter knows where the uncertainty comes from, makes a fair estimate of it, and carries it through to the final answer.



A micrometer screw gauge measures to the nearest 0.01 mm



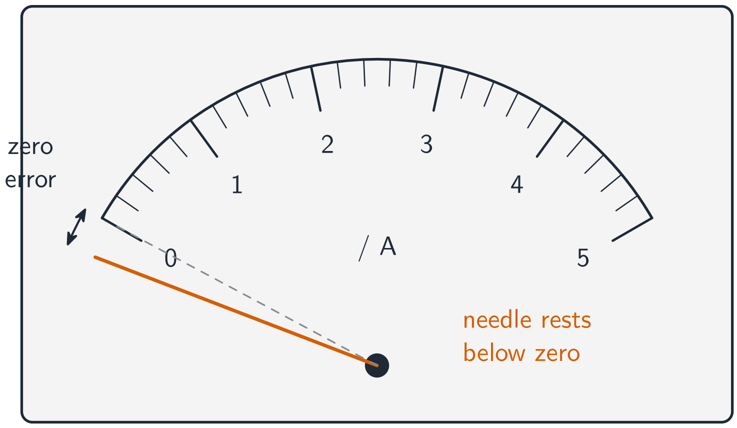
Reading a micrometer: add the main scale reading to the thimble reading

Vernier calipers measure to the nearest 0.1 mm —the sliding scale gives the extra digit

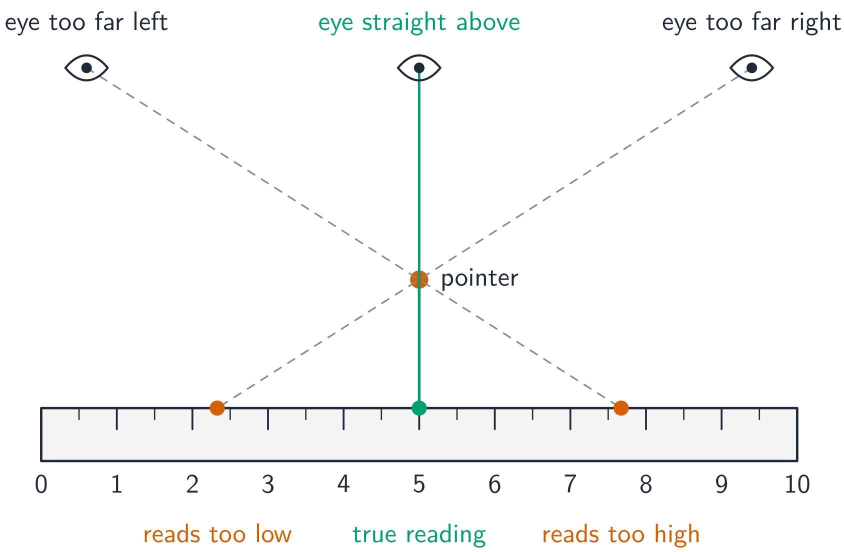
Systematic and random errors

A **systematic error** 系统误差 changes every reading by the same amount, in the same direction. You cannot find it by repeating the measurement. Common causes:

- a **zero error** 零点误差 (the scale does not read zero when the true value is zero)
- a **calibration** 校准 error (the scale itself is wrong)
- **parallax** 视差 (your eye is always to one side of the scale)



An ammeter with a zero error: the needle reads below zero before any current flows



Parallax error: different viewing angles give different scale readings

A systematic error makes the **accuracy** 准确度 worse, but it does not change the **precision** 精密度.

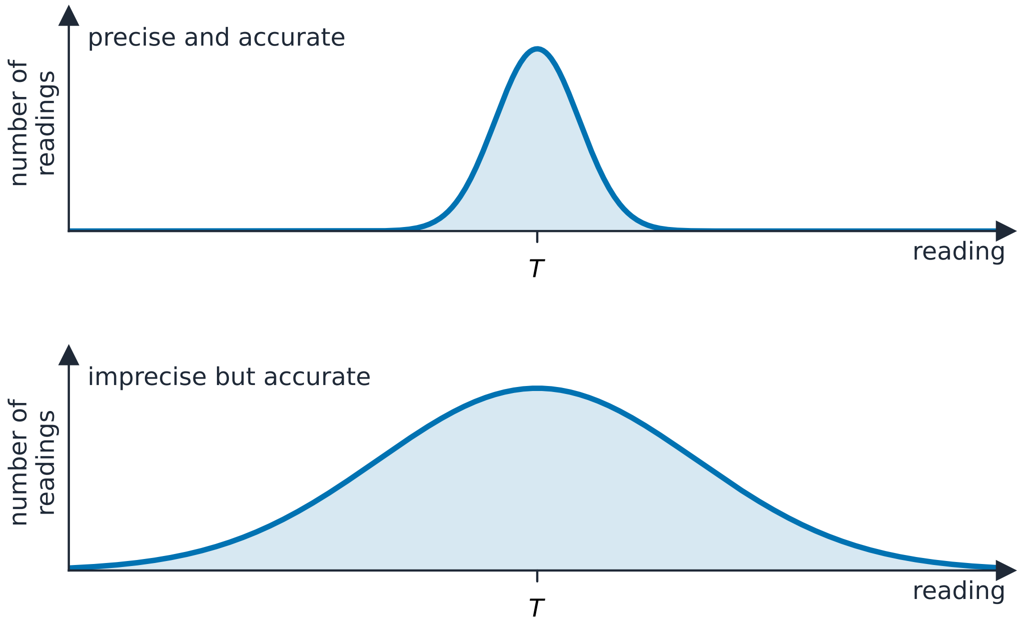
A **random error** 随机误差 makes readings jump above and below the true value, with no pattern. Causes include how carefully you read the scale, changing conditions, and the smallest step the **instrument** 仪器 can show. If you repeat the measurement many times and take the **mean** 平均值 (the average), random errors partly cancel out.

A random error makes the precision worse. But with enough repeats, the mean can still be accurate.

Precision and accuracy

Precision is how close repeated readings are to each other. Precise readings are grouped very close together.

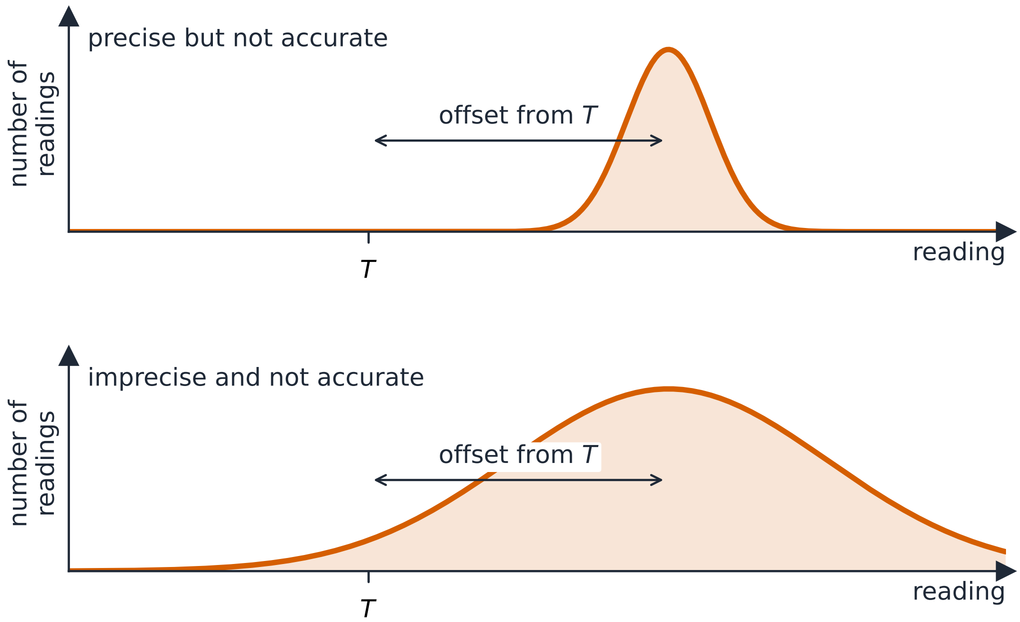
Accuracy is how close a reading (or the mean of several readings) is to the true value.



Precision: how narrow the distribution is around the true value T

A set of readings can be:

- precise and accurate —close together and near the true value
- precise but not accurate —close together, but away from the true value (a systematic error)
- accurate but not precise —spread out, but the mean is near the true value
- neither —spread out and away from the true value



Accuracy: whether the peak of the distribution is centred on the true value T

When a question gives a table of repeated readings, look at the **spread** (precision) and the **mean** (accuracy) separately.

Uncertainty in a derived quantity

A measurement is often written as $x \pm \Delta x$. Here Δx is the **absolute uncertainty** 绝对不确定度. The **percentage uncertainty** 百分比不确定度 is

$$\text{percentage uncertainty in } x = \frac{\Delta x}{|x|} \times 100\%.$$

A **derived quantity** 导出量 is one you calculate from measured values. Its uncertainty is found by simple rules:

- **Adding or subtracting** —add the absolute uncertainties. If $y = a + b$ or $y = a - b$, then $\Delta y = \Delta a + \Delta b$.
- **Multiplying or dividing** —add the percentage uncertainties. If $y = \frac{a \cdot b}{c}$, then

$$\frac{\Delta y}{|y|} = \frac{\Delta a}{|a|} + \frac{\Delta b}{|b|} + \frac{\Delta c}{|c|}.$$

- **Powers** —multiply the percentage uncertainty by the power. If $y = a^n$, then $\frac{\Delta y}{|y|} = |n| \cdot \frac{\Delta a}{|a|}$.

Worked example. A ball's **diameter** 直径 is measured as $d = (5.26 \pm 0.02)$ cm. The **volume** 体积 of a sphere is $V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(d/2)^3$, so $V \propto d^3$ (V depends on d cubed).

The percentage uncertainty in d is

$$\frac{0.02}{5.26} \times 100\% \approx 0.38\%.$$

Because $V \propto d^3$, the percentage uncertainty in V is three times this, about 1.14%. The volume is $\frac{4}{3}\pi(2.63)^3 \approx 76.2$ cm^3. So the absolute uncertainty is $0.0114 \times 76.2 \approx 0.87$ cm^3. The final answer is $V = (76.2 \pm 0.9)$ cm^3.

Significant figures

When you write a calculated quantity, give it the **same number of significant figures** 有效数字 as the least precise measurement you used —usually two or three in this syllabus. Too many significant figures makes the answer look more exact than it really is. Too few loses useful information.

Scalars and vectors

A **scalar** 标量 has size only. A **vector** 矢量 has both size and direction.

Examples from the syllabus:

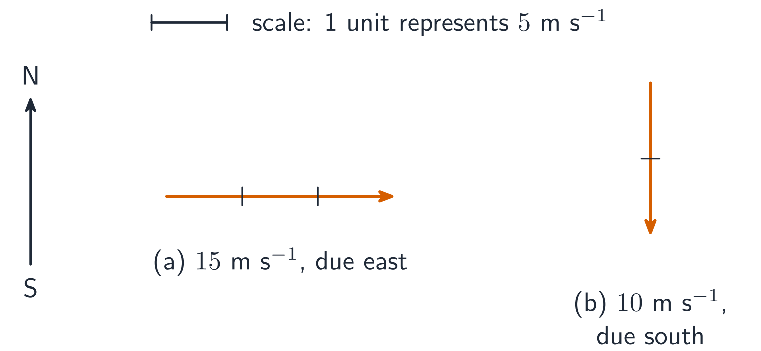
- scalars: mass, time, temperature, energy, work, power, distance, speed, pressure, **density** 密度, **electric charge** 电荷

- vectors: **displacement** 位移, velocity, acceleration, force (including weight), **momentum** 动量

Quick test: if it makes sense to ask "in which direction?", the quantity is a vector. You cannot ask "in which direction is the temperature?", so temperature is a scalar. You can ask "in which direction is the velocity?", so velocity is a vector.

Adding and subtracting vectors

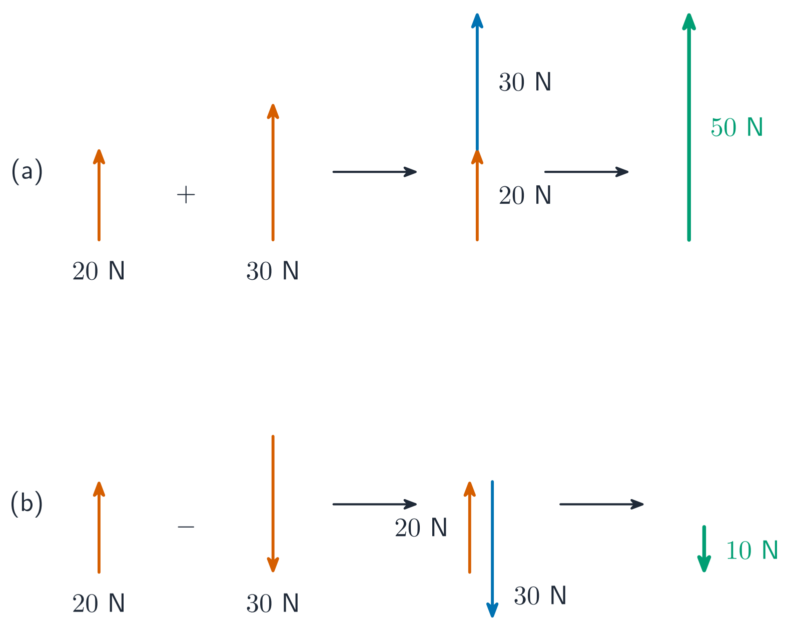
A vector is drawn as an arrow: the **direction** of the arrow gives the direction of the quantity, and the **length** of the arrow (drawn to scale) gives the magnitude.



Vectors represented as arrows drawn to scale

To add two **coplanar** 共面 vectors (vectors in the same flat plane), draw them **tip to tail**. The **resultant** 合矢量 goes from the tail of the first arrow to the tip of the second.

To find $\vec{X} - \vec{Y}$, add the reverse of $\vec{Y}$: $\vec{X} + (-\vec{Y})$. The reverse of $\vec{Y}$ has the same size as $\vec{Y}$ but points the opposite way.



Adding and subtracting parallel vectors

If the two vectors are at right angles (90°), the size of the resultant is

$$|\vec{R}| = \sqrt{X^2 + Y^2},$$

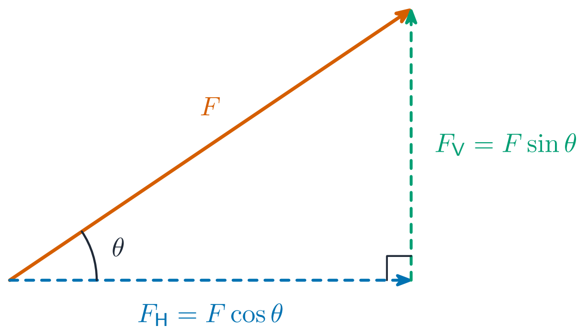
and its direction comes from $\tan\theta = Y/X$.

If two vectors have the same size F with an angle 2α between them, the resultant has size $2F\cos\alpha$ and lies along the line that cuts the angle in half.

Splitting a vector into perpendicular parts

Any vector can be split into two **perpendicular** 垂直 (at right angles) **components** 分量. Usually these are **horizontal** 水平 and **vertical** 竖直, or along and across a surface. For a vector $\vec{v}$ at angle θ to the horizontal:

$$v_{\text{H}} = v\cos\theta, \qquad v_{\text{V}} = v\sin\theta.$$



Resolving a vector into horizontal and vertical components

Choose the directions that make the problem easiest. On a slope (an **inclined plane** 斜面), split the weight into one part along the slope and one part at right angles to it:

$$W_{\parallel} = W\sin\theta, \qquad W_{\perp} = W\cos\theta,$$

where θ is the angle of the slope to the horizontal.

You split a vector into components whenever you need to know how much of it acts in one direction. For example: the part of a force that acts along a slope, or the horizontal and vertical parts of a ball's velocity after it is thrown.