

# Kinematics

## A-Level Physics

### Key definitions



*A speedometer shows speed: the distance travelled per unit time.*

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These five quantities come up in almost every **kinematics** 运动学 question. Learn the exact words —the examiner gives marks for precise wording.

- **distance** 距离—the total length of the path travelled. A **scalar** 标量.
- **displacement** 位移—the straight-line distance from the start to the end, with a direction. A **vector** 矢量.
- **speed** 速率—the rate of change of distance with time. A scalar.
- **velocity** 速度—the rate of change of displacement with time. A vector.
- **acceleration** 加速度—the rate of change of velocity with time. A vector.

The unit of speed and velocity is  $\text{m s}^{-1}$ ; the unit of acceleration is  $\text{m s}^{-2}$ .

A common mistake: **deceleration** 减速度 just means acceleration in the opposite direction to the velocity. It is not a separate quantity.

# Motion graphs



*A high-speed train: its motion can be shown on a distance-time graph.*

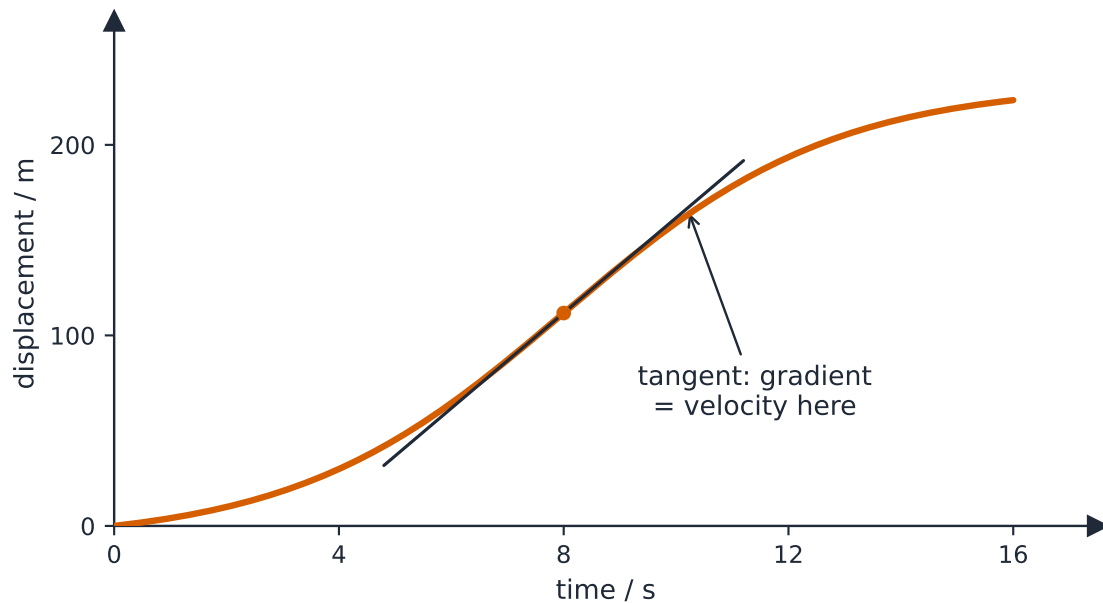
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Many marks come from reading or drawing motion graphs. Two graphs matter.

## Displacement–time graph

The **gradient** 斜率 (steepness) of a displacement–time graph at a point gives the velocity at that moment.

- flat line → the object is at rest.
- straight sloping line → constant velocity (gradient = velocity).
- curved line → changing velocity. Draw a **tangent** 切线 at the point and find its gradient.



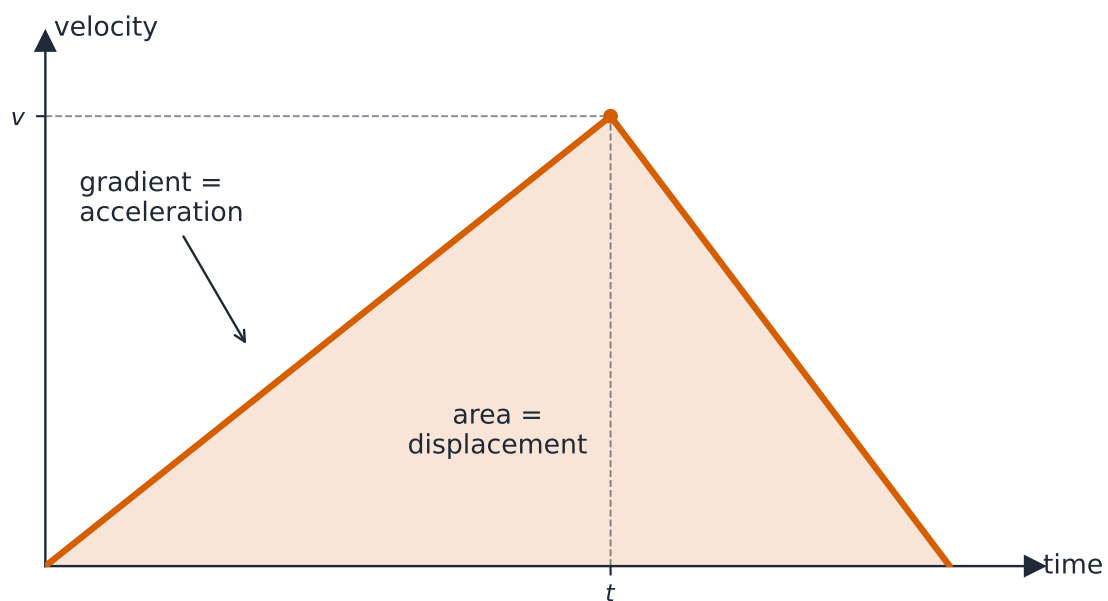
*Displacement–time graph of a car on a test track*

## Velocity–time graph

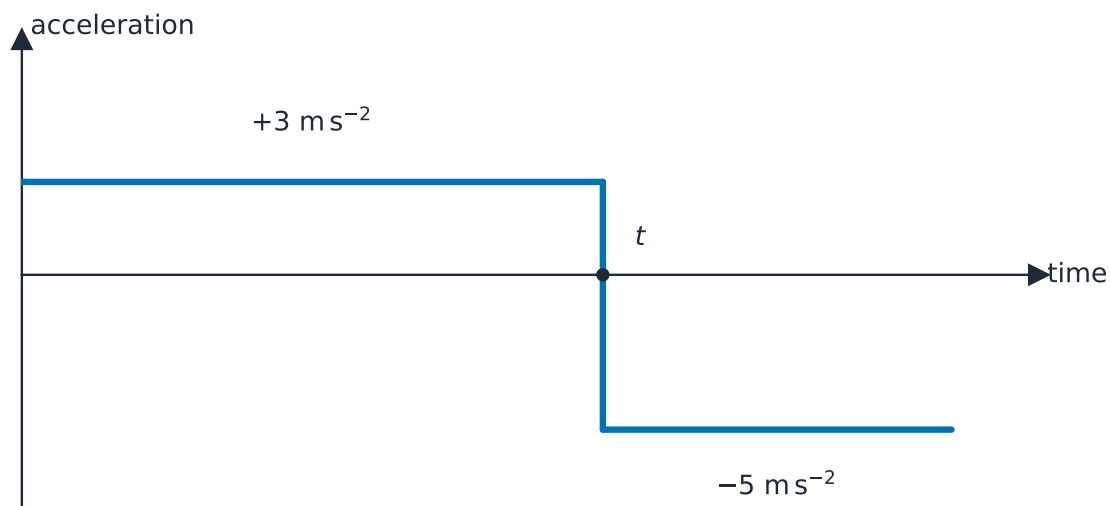
The gradient of a velocity–time graph gives the acceleration at that moment.

The **area** between the line and the time axis gives the displacement in that time.

- flat line → constant velocity (zero acceleration).
- straight sloping line → **uniform acceleration** 匀加速 (constant acceleration).
- curved line → changing acceleration.
- area above the time axis is positive displacement; area below is negative (the object moved backwards).



*Velocity–time graph — gradient gives acceleration, area gives displacement*



*Acceleration–time graph derived from the same motion*

To find the displacement, split the area into triangles and rectangles, or count grid squares. Area of a triangle is  $\frac{1}{2} \times \text{base} \times \text{height}$ ; area of a rectangle is  $\text{base} \times \text{height}$ .

## The four SUVAT equations

For motion in a straight line with uniform acceleration, we use five symbols: starting velocity  $u$ , final velocity  $v$ , acceleration  $a$ , displacement  $s$ , and time  $t$ . Four equations link them:

$$v = u + at$$

$$s = ut + \frac{1}{2}at^2$$

$$s = \frac{1}{2}(u + v)t$$

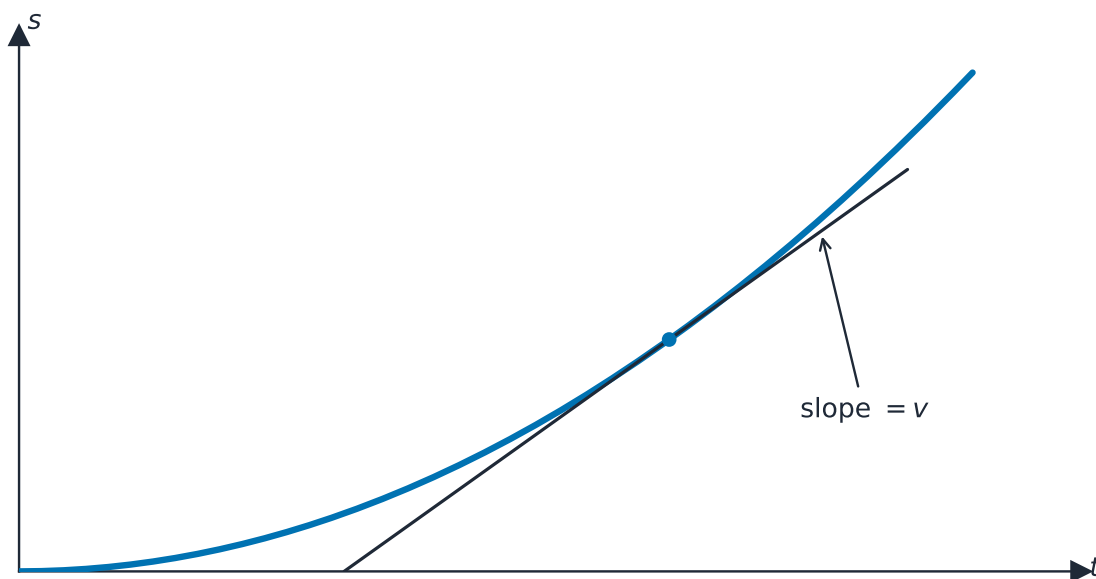
$$v^2 = u^2 + 2as$$

Each equation uses four of the five symbols. To pick the right one: write down what you know and what you want, then choose the equation with exactly those four.

## Where the SUVAT equations come from

You should be able to get these from the definitions of velocity and acceleration:

- $v = u + at$  comes from  $a = (v - u)/t$ , the gradient of the line.
- $s = \frac{1}{2}(u + v)t$  is the area under the line — a **trapezium** 梯形 with parallel sides  $u$  and  $v$  and width  $t$ .
- $s = ut + \frac{1}{2}at^2$  comes from putting  $v = u + at$  into the area.
- $v^2 = u^2 + 2as$  comes from removing  $t$  from the first two.



*Displacement–time graph for uniform acceleration —the slope at any point equals the instantaneous velocity*

If a question asks "which equation can be found using only the gradient of a velocity–time graph?", the answer is  $v = u + at$  (the gradient is the acceleration).

## Choosing a positive direction

Pick a positive direction at the start and keep it. Anything pointing the other way gets a minus sign. For a ball thrown straight up, if "up" is positive:  $u$  is positive,  $a = -g$  (**gravity** 重力 pulls down), and at the highest point the displacement is positive but the velocity is zero.

## Free fall under gravity

When **air resistance** 空气阻力 can be ignored, an object in **free fall** 自由落体 has a constant acceleration  $g \approx 9.81 \text{ m s}^{-2}$  downwards. This is the same for every mass.

For a ball dropped from rest and falling a distance  $h$ :

$$h = \frac{1}{2}gt^2, \quad v = gt, \quad v^2 = 2gh.$$

For a ball thrown straight up with speed  $u$ :

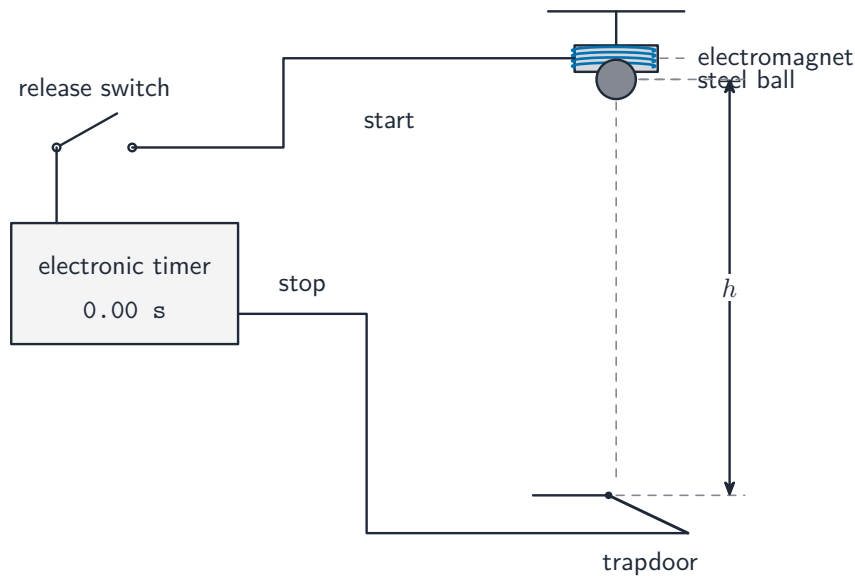
- greatest height: put  $v = 0$  in  $v^2 = u^2 - 2gh$ , giving  $h = u^2/(2g)$ .
- time to reach the top: put  $v = 0$  in  $v = u - gt$ , giving  $t = u/g$ .
- total time to fall back to the start height:  $2u/g$  (the motion is **symmetric** 对称).

## Experiment to find $g$

A common method: drop an object from rest, then measure the distance  $h$  it falls and the time  $t$  it takes. Then

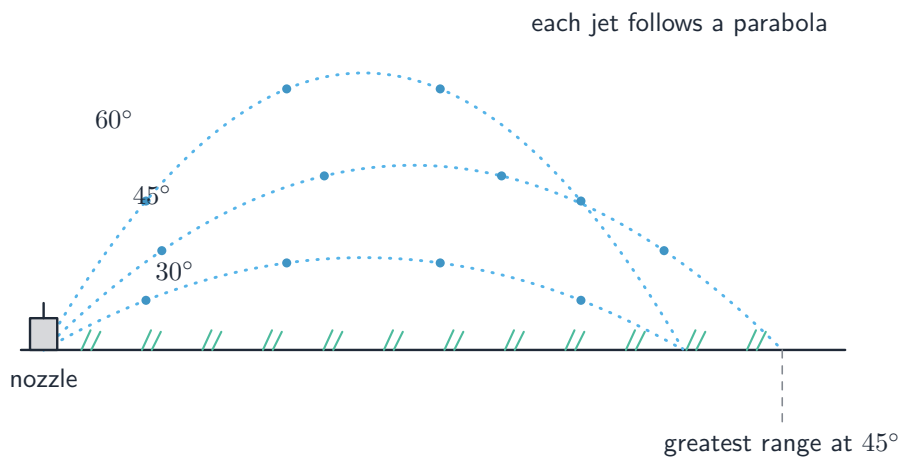
$$g = \frac{2h}{t^2}.$$

Repeat for several heights and plot  $h$  against  $t^2$ . The gradient of the best straight line is  $g/2$ , so  $g$  is twice the gradient. Repeating reduces **random error** 随机误差. An electronic timer —using **light gates** 光电门, or a switch the ball hits —removes **reaction-time** 反应时间 error.



*Experimental set-up for measuring the acceleration due to free fall*

## Motion in two directions



*Water jets from a sprinkler trace parabola paths —a real example of projectile motion*

When an object moves at constant velocity in one direction (say **horizontal** 水平) and speeds up in a direction at right angles to it (say **vertical** 竖直, under gravity), the two motions do not affect each other. Treat each direction on its own, with its own SUVAT equation.

## Horizontal throw

An object thrown horizontally with speed  $u_H$  from height  $h$ , with air resistance ignored:

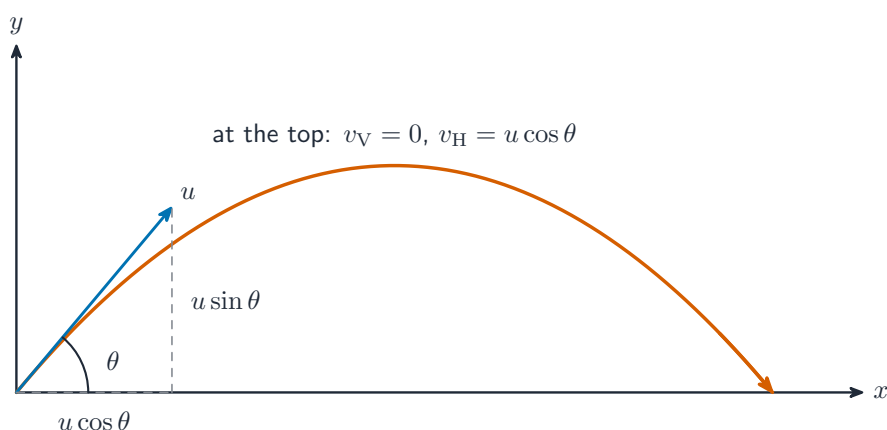
- **horizontal**: constant velocity  $u_H$ . After time  $t$ , the horizontal distance is  $x = u_H t$ .
- **vertical**: starts from rest and speeds up downwards at  $g$ . After time  $t$ , it has fallen  $y = \frac{1}{2}gt^2$  and has vertical velocity  $v_V = gt$ .

The time to reach the ground depends **only** on the height  $h$ , not on  $u_H$ . Solve  $h = \frac{1}{2}gt^2$  for  $t$ ; then the horizontal **range** 射程 is  $u_H t$ .

The horizontal-velocity graph is a flat line at  $u_H$ . The vertical-velocity graph is a straight line from the origin with gradient  $g$ .

## Projectile at an angle

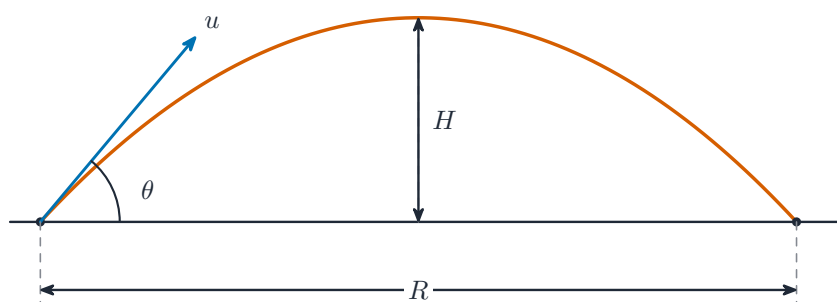
A **projectile** 抛体 thrown at speed  $u$  at angle  $\theta$  above the horizontal:



*Projectile launched at angle  $\theta$  —horizontal and vertical motions are independent*

- horizontal **component** 分量 of the starting velocity:  $u_H = u \cos \theta$  (stays constant during the flight).
- vertical component of the starting velocity:  $u_V = u \sin \theta$  (gets smaller, becomes zero at the top, then grows downwards).

At the highest point,  $v_V = 0$ , but  $v_H$  is still  $u \cos \theta$ . The time to the top is  $t_{\text{up}} = u \sin \theta / g$ ; the total flight time (back to the start height) is  $2t_{\text{up}}$ .



*Range  $R$  of a projectile launched from and landing on level ground*

## Bouncing ball

When a ball bounces, its velocity–time graph is a set of straight sloping lines (constant  $g$ ) with a sudden jump at each bounce (the velocity flips direction, and gets smaller if some **energy** 能量 is lost). Add up the times and the distances across the bounces.

## Two objects meeting

When two objects move along the same line in different ways, write a displacement equation for each. Use the same start time and the same positive direction. Then set the two displacements equal (or set their difference to a given gap).

For a goods train at constant velocity  $u_G$  and an express train starting from rest with acceleration  $a$ , both passing the same point at  $t = 0$ :

$$s_G = u_G t, \quad s_E = \frac{1}{2} a t^2.$$

They are level again when  $s_G = s_E$ , giving  $t = 2u_G/a$ .

## Tips for solving problems

1. **Draw a diagram** and mark the positive direction.
2. **List the SUVAT symbols** with their known and unknown values, including signs.
3. **Choose the SUVAT equation** with exactly the four symbols you have, plus the one you want.
4. For projectile motion, **split into horizontal and vertical** SUVAT problems, linked only by the time  $t$ .
5. Always **check the units** of your answer, and that its size is sensible.