Forces, density and pressure

A-Level Physics

Turning effects of forces

Centre of gravity

The **weight** 重力 of a large object can be treated as acting at one single point, called the **centre of gravity** 重心 (the same as the **centre of mass** 质心 in a uniform gravitational field). For a uniform, regular shape —a rectangle, a sphere, a uniform rod —the centre of gravity is at the middle.

When you draw a **free-body diagram** 受力图, always put the weight arrow at the centre of gravity.

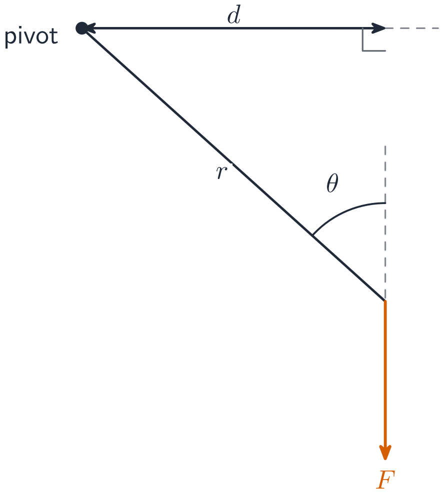
Moment of a force

The **moment** 力矩 of a force about a point is

$$M = F \cdot d,$$

where F is the size of the **force** 力 and d is the **perpendicular distance** 垂直距离 from the point to the **line of action** 作用线 of the force. Unit: N m.

If the force acts at angle θ to a **lever arm** 力臂 of length r from the **pivot** 支点, then $d = r \sin\theta$, so $M = Fr\sin\theta$. Only the part of the force **perpendicular** 垂直 to the lever arm makes it turn.



The moment depends on the perpendicular distance d from the pivot to the line of action of the force

A moment is either **clockwise** 顺时针 or **anticlockwise** 逆时针 about the chosen point.

Couple and torque

A **couple** 力偶 is a pair of forces that are:

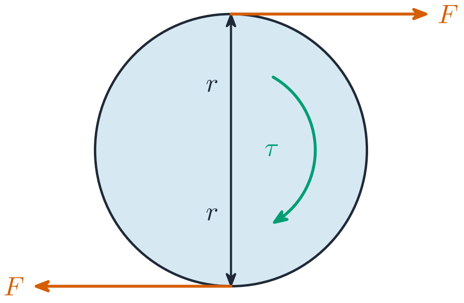
- **equal in size**,
- **opposite in direction**,
- with their lines of action a perpendicular distance apart.

A couple makes the body **turn only** —its resultant force is zero, so it gives no straight-line acceleration.

The **torque** 力偶矩 of a couple is the turning effect it makes:

$$\tau = F \cdot d,$$

where F is the size of one force and d is the perpendicular distance between the two lines of action. Unit: N m. The torque is the same about any point —a special property of couples.



A couple: two equal and opposite forces, a distance apart, producing a torque

A common multiple-choice trap: two equal forces in the **same** direction are not a couple (they have a resultant force and cause **translation** 平动). A couple needs equal size **and** opposite direction.

Equilibrium of forces

Conditions for equilibrium

A body is in **equilibrium** 平衡 when:

1. the **resultant force** 合力 is zero (no straight-line acceleration), AND
2. the resultant moment about any point is zero (no **angular acceleration** 角加速度).

Both must hold. A body with no resultant force can still be turning; a body with no resultant moment can still be moving in a straight line.

Principle of moments

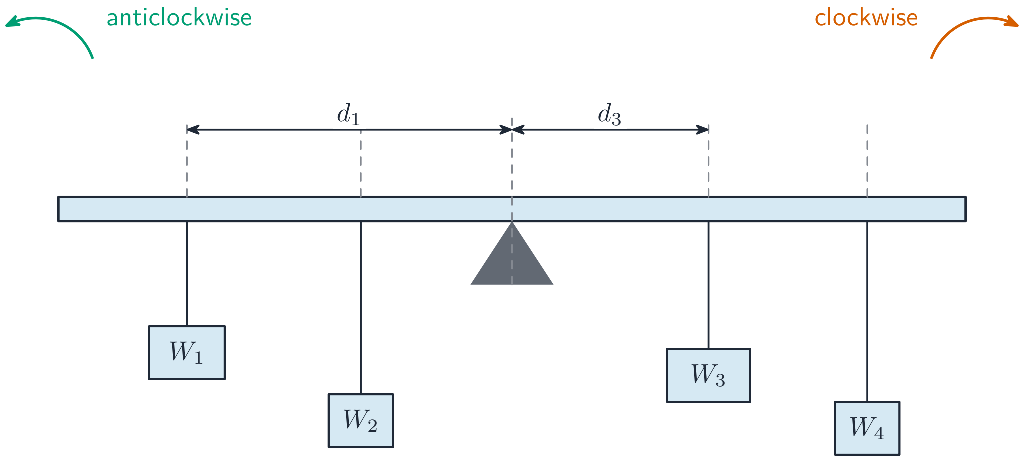
For a body that is not turning, **the total clockwise moment about any point equals the total anticlockwise moment about the same point**. This is the **principle of**

moments 力矩原理.

To solve a balance problem:

1. Choose a pivot —usually where an unknown force acts, so that force drops out (its distance is zero).
2. List every force and its perpendicular distance from the pivot.
3. Set $\sum M_{\text{clockwise}} = \sum M_{\text{anticlockwise}}$.
4. Use $\sum F = 0$ if you need a second equation.

A ruler balanced on a pivot with masses on each side is solved this way. For a heavy uniform rod, remember to include its weight acting at its centre of gravity.



Weights on a rule balanced at a pivot —used to test the principle of moments

Vector triangle

Three forces in the same plane that are in equilibrium can be drawn as a **closed vector triangle** 矢量三角形—drawn tip-to-tail, the three arrows come back to the start. This is a drawing method instead of splitting into **components** 分量.

Use the **sine rule** 正弦定理 or the **cosine rule** 余弦定理 on the triangle to find unknown sizes or directions, or draw the triangle to scale on graph paper.

You can also split each force into **horizontal** 水平 and **vertical** 竖直 components and set $\sum F_x = 0$ and $\sum F_y = 0$.

Density

An iceberg floats with most of its volume hidden: ice is slightly less dense than water.

Image: IlyaHaykinson, CC BY 2.0 (commons.wikimedia.org)

Density 密度 is the mass per unit volume:

$$\rho = \frac{m}{V}.$$

Unit: kg m^{-3} (or g cm^{-3}; 1 g cm^{-3} = 1000 kg m^{-3}). Density is a **scalar** 标量.

Some useful densities to know:

- water: 1000 kg m^{-3}
- air at room conditions: ~ 1.2 kg m^{-3}
- iron / steel: ~ 7800 kg m^{-3}

Pressure

Pressure 压强 is the force per unit area, where the force acts at right angles to the surface:

$$p = \frac{F}{A}.$$

Unit: Pa = N m^{-2}. Pressure is a scalar.

A precision aneroid barometer measures atmospheric pressure

Image: Fischer, Product image (weatherscientific.com)

Hydrostatic pressure

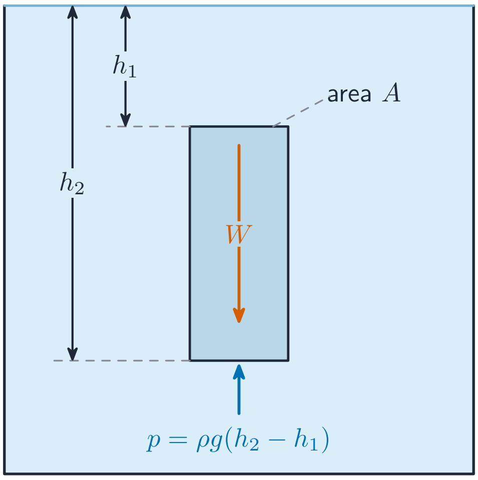
Take a column of **fluid** 流体 with density ρ, **cross-sectional area** 横截面积 A and height Δh. Its weight is

$$W = mg = (\rho \cdot A \cdot \Delta h) \cdot g.$$

This weight presses down on the area A at the bottom, so the extra pressure at the bottom compared with the top is

$$\Delta p = \frac{W}{A} = \rho g \Delta h.$$

This is the **hydrostatic pressure** 流体静压强 equation. It depends only on the density of the fluid and the **depth** 深度—the shape of the container does not matter.

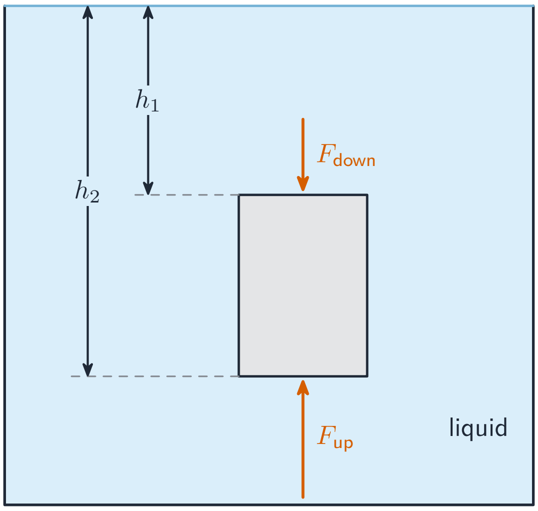


A column of liquid of area A: its weight sets the extra pressure at the depth below

For a submarine at depth h below the surface, the pressure from the water is $\rho_{\text{seawater}}\, g\, h$. For the total pressure, add the **atmospheric pressure** 大气压强 at the surface (about 1.0×10^5 Pa).

Upthrust and Archimedes' principle

When an object is **submerged** 浸没 in a fluid, the pressure at the bottom of the object is greater than the pressure at the top (by $\rho g \Delta h$, where Δh is the object's height). This difference gives a net upward force called the **upthrust** 浮力.



Upthrust arises because the pressure on the bottom of the object is greater than on the top

For an object of volume V (the volume of fluid **displaced** 排开), the upthrust is

$$F_{\text{upthrust}} = \rho_{\text{fluid}}\, g\, V.$$

This is **Archimedes' principle** 阿基米德原理: the upthrust on a body in a fluid equals the weight of the fluid it pushes aside.

For a fully submerged object, V is its full volume. For a **floating** 漂浮 object, V is only the volume below the surface —the object floats when the upthrust on the part below the surface equals its weight.

Force balance with upthrust

A block held under water by a string tied to the bottom of the container is in equilibrium under three vertical forces: weight (down), **tension** 张力 (down), upthrust (up). Set $F_{\text{upthrust}} = W + T$ to find the tension.

A submerged block hanging from a **newton meter** 弹簧测力计 reads less than its weight in air, because of the upthrust: reading $= W - F_{\text{upthrust}}$.

The upthrust depends on the fluid density and the displaced volume, not on the object's material or depth (for an **incompressible** 不可压缩 fluid). On a planet with smaller g, the upthrust is smaller in the same ratio as the weight, so a floating object still floats with the same fraction below the surface.