Work, energy and power

A-Level Physics

Work, energy and power

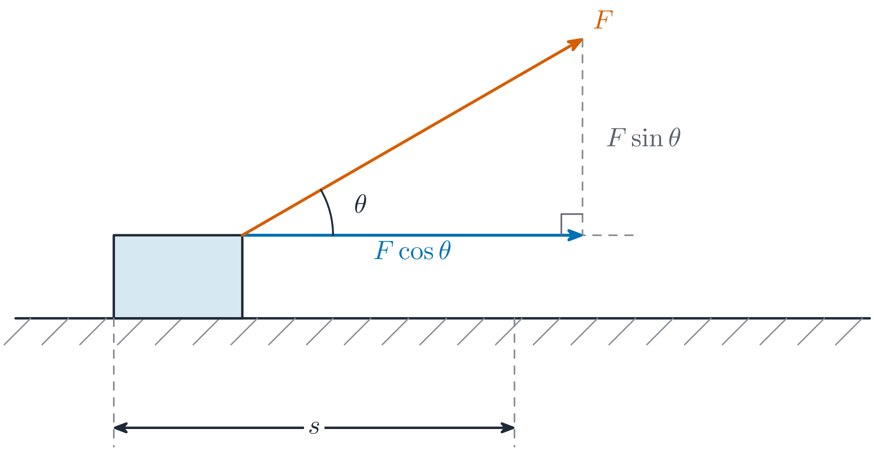
Wind turbines transfer the kinetic energy of the wind into electrical energy.

Work done by a force

Work 功 is done when a force moves its point of contact along the line of the force. The work done by a constant force F that causes a **displacement** 位移 s is

$$W = F \cdot s \cdot \cos\theta,$$

where θ is the angle between the force and the displacement. Only the **component** 分量 of the force along the displacement does work.

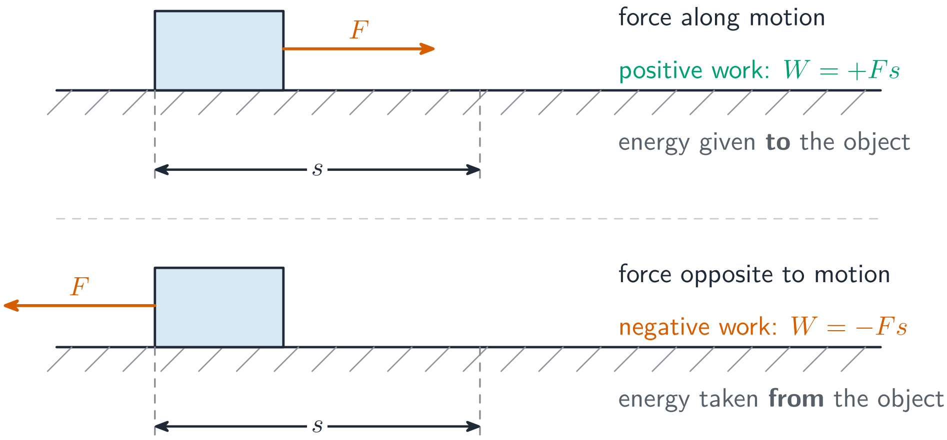


Only the component of the force along the displacement ($F\cos\theta$) does work

Unit: J = N m. Work is a **scalar** 标量.

Special cases:

- force in the same direction as the motion ($\theta = 0$): $W = Fs$, positive work, **energy** 能量 given **to** the object.
- force at right angles to the motion ($\theta = 90°$): $W = 0$. The **normal contact force** 支持力 on a car on a flat road does no work.
- force opposite to the motion ($\theta = 180°$): $W = -Fs$, negative work, energy taken **from** the object (for example **friction** 摩擦力).



Positive work ($W = +Fs$): force along the motion (top). Negative work ($W = -Fs$): force opposite to the motion, e.g. friction (bottom)

For an object moving up a slope at angle α to the **horizontal** 水平, the work done against gravity in rising a height h is mgh, while the work done by a horizontal push over the slope length L uses $\cos\alpha$.

Conservation of energy

Energy is **never made or destroyed** —it only changes from one form to another, or moves from one object to another. In a **closed system** 封闭系统, the total energy stays constant. This is **conservation of energy** 能量守恒.

When you write an energy equation, list every form the energy starts as and ends as. Common forms in this syllabus: kinetic, gravitational potential, **elastic potential energy** 弹性势能, **electrical** 电能, thermal, sound, **chemical energy** 化学能.

A ball rolling down a **frictionless** 无摩擦 **ramp** 斜坡 turns **gravitational potential energy** 重力势能 into **kinetic energy** 动能: $mgh = \frac{1}{2}mv^2$, so $v = \sqrt{2gh}$. With friction, some of this energy becomes **thermal energy** 热能 of the ramp and the air.

Efficiency

The **efficiency** 效率 of a system is

$$\text{efficiency} = \frac{\text{useful energy output}}{\text{total energy input}} \times 100\%.$$

The same idea with power:

$$\text{efficiency} = \frac{\text{useful power output}}{\text{total power input}} \times 100\%.$$

Efficiency is always less than 100% in a real system, because some input energy becomes "useless" forms —usually thermal energy.

For an electric motor lifting a load with efficiency η at **voltage** 电压 V and **current** 电流 I, the useful output power is ηVI. From this you can find a force, a lifting speed, or a **tension** 张力.

Power

Power 功率 is the rate of doing work, or the rate of transferring energy:

$$P = \frac{W}{t} = \frac{\Delta E}{\Delta t}.$$

Unit: $\mathrm{W} = \mathrm{J\ s^{-1}}$. Power is a scalar.

Power, force and velocity

For an object moving at **velocity** 速度 v with a force F along the direction of motion, in a short time Δt the displacement is $v\,\Delta t$ and the work done is $Fv\,\Delta t$. Dividing by Δt:

$$P = Fv.$$

This is one of the most useful results in mechanics.

- For a car at constant velocity v on a flat road, the engine power must balance the total resistive force: $P = F_{\text{resist}} \cdot v$. If the **drag** 阻力 grows with v^2, doubling the speed roughly **quadruples** the power needed.
- For lifting a **weight** 重力 mg straight up at constant speed v, the useful output power is $P = mg \cdot v$.
- For an aircraft hovering at a fixed height, the lift force equals the weight, and a large power is needed because air must be pushed downwards all the time.

Gravitational potential energy

In a **uniform** gravitational field (close to a planet's surface), the change in gravitational potential energy of **mass** 质量 m rising or falling through a height Δh is

$$\Delta E_{\text{P}} = mg\Delta h.$$

Where it comes from

The work done **against** gravity to raise a mass m slowly (no change in kinetic energy) through height Δh equals the gravitational potential energy gained:

- the gravitational force on the mass is mg downwards,
- the force needed to lift it slowly is mg upwards,
- the work done by this force is $W = F \cdot s = mg \cdot \Delta h$,
- this work becomes ΔE_{P}.

So $\Delta E_{\mathrm{P}} = mg\Delta h$. To use it you need m and Δh (and g). You do not need speed or time.

Kinetic energy

The kinetic energy of an object of mass m moving at speed v is

$$E_{\mathrm{K}} = \tfrac{1}{2}mv^2.$$

Where it comes from

Apply a resultant force F to a mass m that starts at rest. It speeds up evenly from 0 to v over a displacement s. From $v^2 = u^2 + 2as$ with $u = 0$,

$$s = \frac{v^2}{2a}.$$

The work done on the mass is

$$W = F \cdot s = ma \cdot \frac{v^2}{2a} = \tfrac{1}{2}mv^2.$$

All this work becomes kinetic energy, so $E_{\mathrm{K}} = \tfrac{1}{2}mv^2$.

Kinetic energy and momentum

Combining $p = mv$ and $E_{\mathrm{K}} = \tfrac{1}{2}mv^2$:

$$E_{\mathrm{K}} = \frac{p^2}{2m}.$$

This is handy when the **momentum** 动量 is given but not the velocity. For a momentum change from p_1 to p_2 at constant mass, the change in kinetic energy is $(p_2^2 - p_1^2)/(2m)$.

Using energy methods

A useful plan for problems that mix forces and energy:

1. **Find the start and end states.** Write the kinetic and potential energies in each.

2. **List any work done by outside forces** (friction, a push). Friction usually takes energy out; a push can add it.
3. **Conservation of energy**: $E_{\text{start}} + W_{\text{in}} = E_{\text{end}} + W_{\text{lost as heat etc.}}$.

Examples:

- A box pushed at **constant velocity** up a ramp of length L rising by h: E_{K} does not change, so the work done by the push goes into ΔE_{P} plus the work done against friction.
- A block sliding into a **spring** 弹簧 with kinetic energy E_{K} on a frictionless surface: at greatest **compression** 压缩 x, all the kinetic energy has become elastic potential energy $\frac{1}{2}kx^2$ (where k is the **spring constant** 劲度系数).
- A ball dropped from height h_1 that bounces to height h_2: the ratio h_2/h_1 is the fraction of mechanical energy kept, $h_2/h_1 = (v_{\text{up}}/v_{\text{down}})^2$.
- A **projectile** 抛体 thrown to the same height at different angles: the final speed is the same (only the height matters); use components to get its direction.