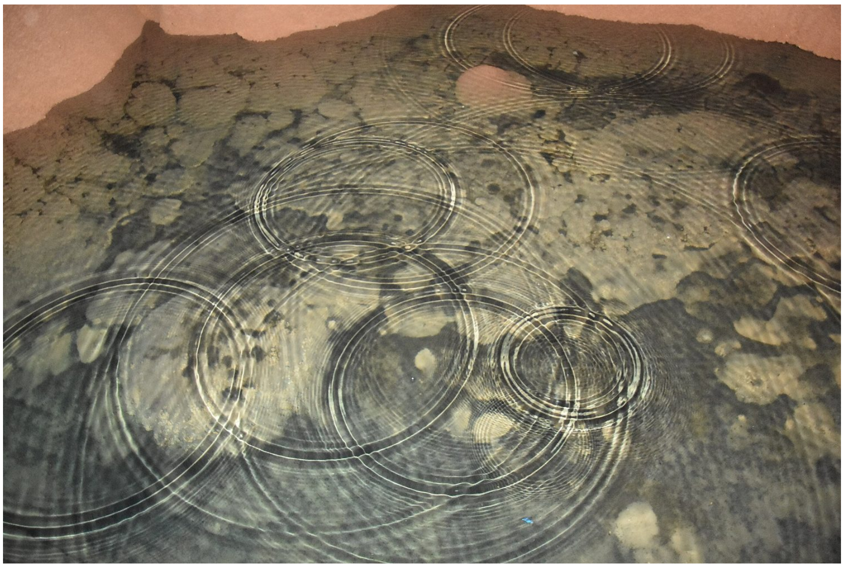
Waves

A-Level Physics

Progressive waves



Ripples spreading on water are progressive waves that carry energy outward.

Image: Georgios Orfeas Katsouris, CC BY 4.0 (commons.wikimedia.org)

A **wave** 波 carries **energy** 能量 from one place to another without moving matter overall. The particles of the **medium** 介质 **oscillate** 振动 about fixed rest positions; only the disturbance (and its energy) **propagates** 传播. Examples: a **transverse wave** 横波 on a rope, a **longitudinal wave** 纵波 on a slinky spring, ripples on water, and sound in air. A wave that travels and carries energy is a **progressive wave** 行波.

Key terms

- **displacement** 位移 y —how far a particle has moved from its rest position at a moment. A **vector** 矢量.
- **amplitude** 振幅 A —the largest displacement from the rest position.
- **wavelength** 波长 λ —the shortest distance along the wave between two points that move **in phase** 同相 (for example, two next-door **crests** 波峰).
- **period** 周期 T —the time for one full oscillation of a particle.
- **frequency** 频率 f —the number of full oscillations per second; $f = 1/T$. Unit: **hertz** 赫兹, Hz.
- **speed** 速率 v —how fast a crest travels along the medium.
- **phase difference** 相位差—the fraction of a cycle by which one oscillation leads or lags another. Given in **radians** 弧度 (a full cycle is 2π) or degrees (a full cycle is 360°).

Two points one wavelength apart are in phase (phase difference 0 or 2π). Two points half a wavelength apart are exactly out of phase (phase difference π).

The wave equation

In one period T, the wave moves forward by one wavelength λ. So speed = distance/time = $\lambda/T = \lambda f$:

$$v = f\lambda.$$

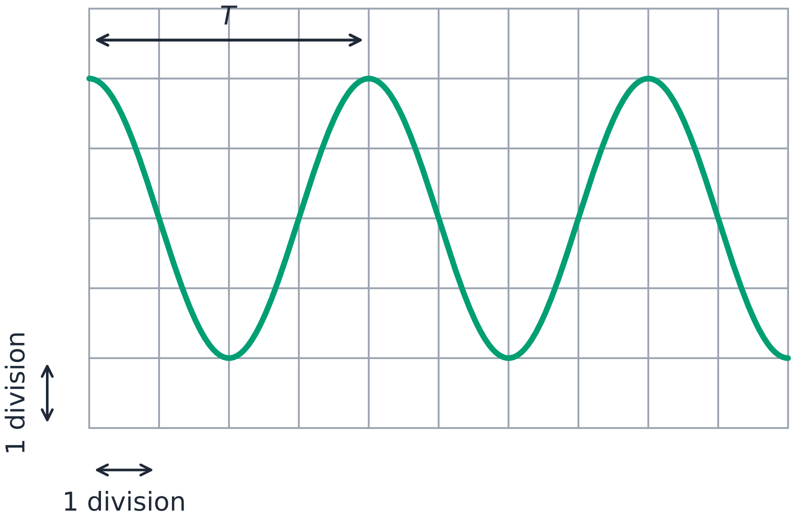
This comes straight from the definitions of speed, frequency and wavelength, and works for every progressive wave.

Reading a CRO trace

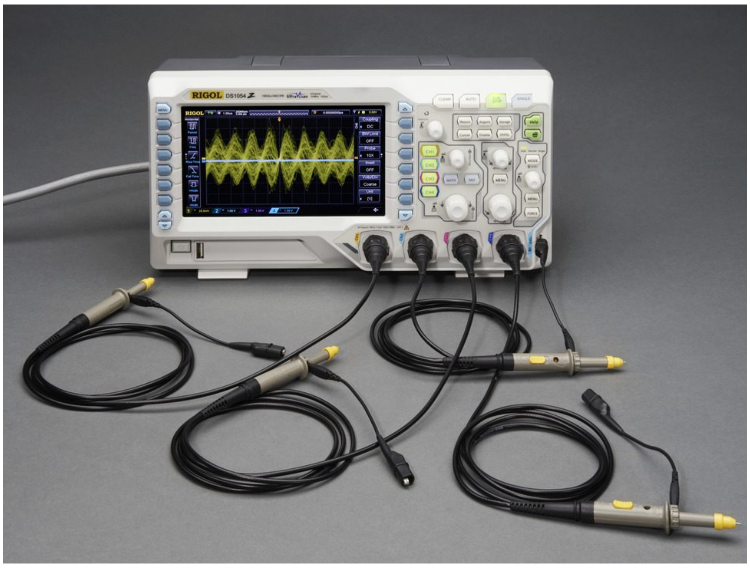
A **cathode-ray oscilloscope** 示波器 (CRO) draws a voltage signal —for sound, the output of a microphone —against time. Two controls matter:

- **time-base** 时基 (seconds per division across): turns horizontal distance on the screen into time. Read the period T as the distance between two next-door peaks, then $f = 1/T$.
- **y-gain** 垂直增益 (volts per division up): turns vertical distance into voltage. The amplitude in volts is the peak height from the centre line.

If the time-base is 5 ms/div and one full cycle takes 4 divisions, then $T = 4 \times 5$ ms = 20 ms and $f = 50$ Hz.



Reading the period T from a CRO trace using the grid and time-base



A real oscilloscope: the grid lets you read off the period and the amplitude

Image: Rigol, Product image (www.adafruit.com)

Intensity of a wave

A wave carries energy. The **intensity** 强度 at a point is the **power** 功率 passing through unit area at right angles to the direction of travel:

$$I = \frac{P}{A}.$$

Unit: W m^{-2}.

Intensity is proportional to the square of the amplitude:

$$I \propto A^2.$$

For a **point source** 点源 sending out energy equally in all directions, the **wavefronts** 波前 are spheres; the surface area at distance r is $4\pi r^2$, so

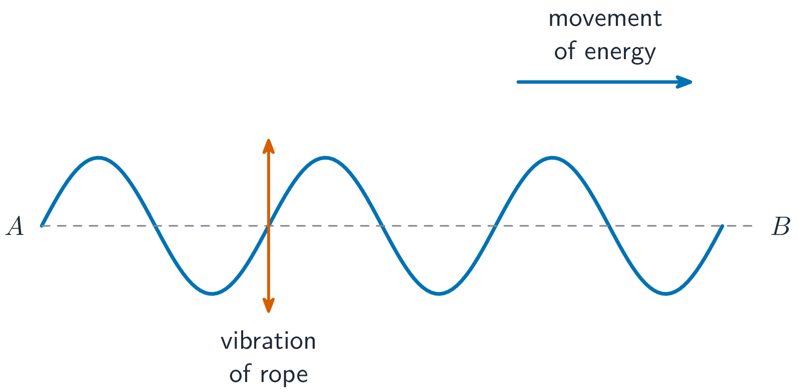
$$I = \frac{P}{4\pi r^2}, \qquad I \propto \frac{1}{r^2}.$$

Doubling the distance cuts the intensity to a quarter, which means the amplitude is halved (since $I \propto A^2$).

Transverse and longitudinal waves

Transverse waves

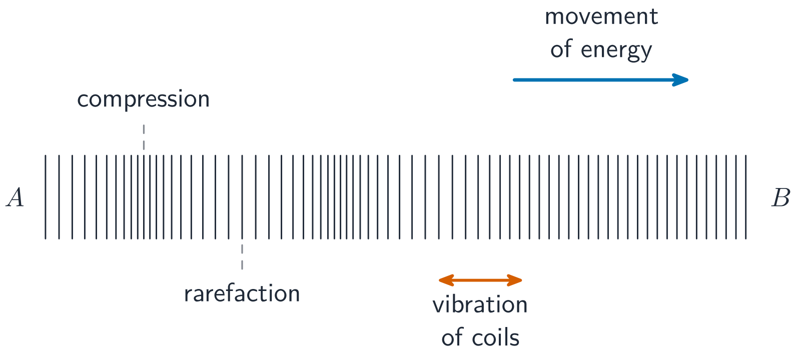
The particles oscillate **perpendicular** 垂直 to the direction the energy travels. A wave on a rope, all electromagnetic waves, and S-waves in the Earth are transverse.



Transverse wave on a rope

Longitudinal waves

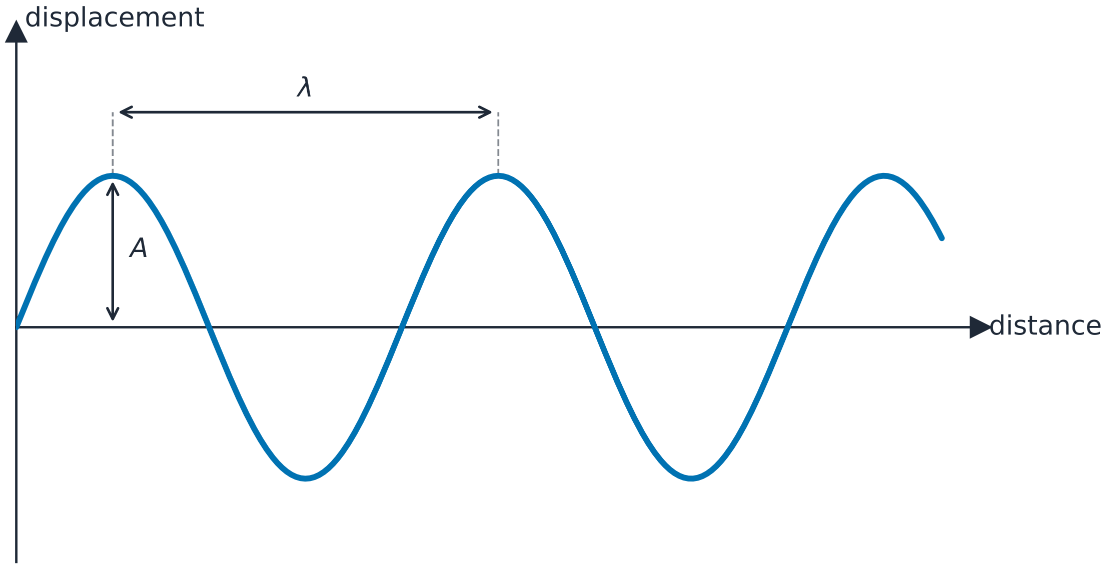
The particles oscillate **parallel** to the direction the energy travels. Sound in any medium, P-waves in the Earth, and the squashes on a slinky are longitudinal. The wave is made of **compressions** 压缩 (higher pressure, particles close together) and **rarefactions** 稀疏 (lower pressure, particles spread out).



Longitudinal wave on a slinky spring

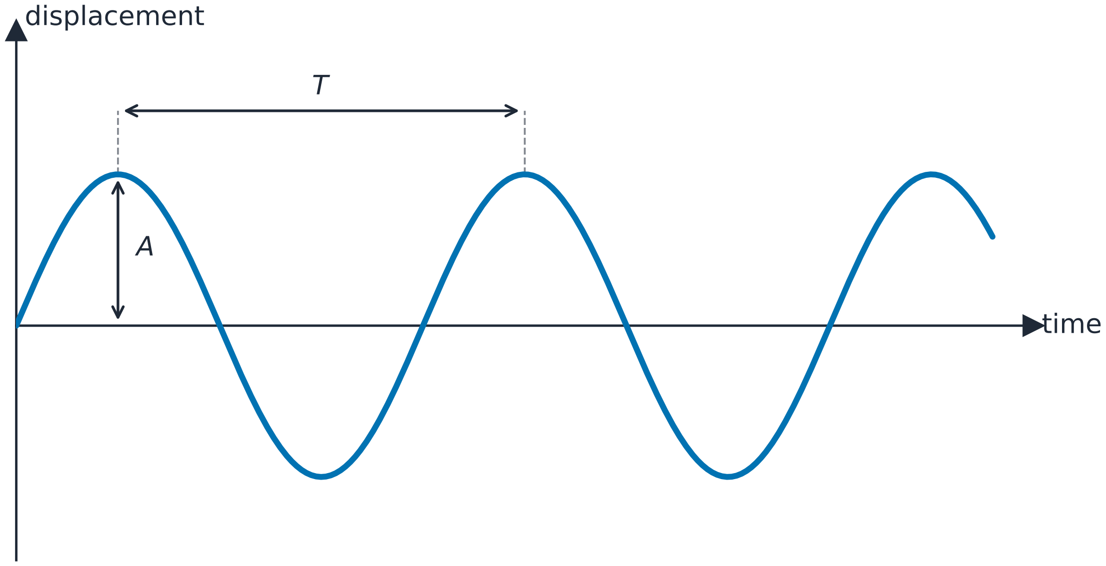
Graphs of waves

A graph of **particle displacement against position** at one moment looks like a **sine curve** 正弦曲线 for both kinds of wave. The difference: for a transverse wave the displacement axis is the real sideways displacement; for a longitudinal wave it is the small back-and-forth displacement along the direction of travel (positive one way, negative the other).



A displacement–distance graph shows the wave's amplitude and wavelength

A graph of **particle displacement against time** at one point in space is also a sine curve for both kinds. Read the period T from this graph.

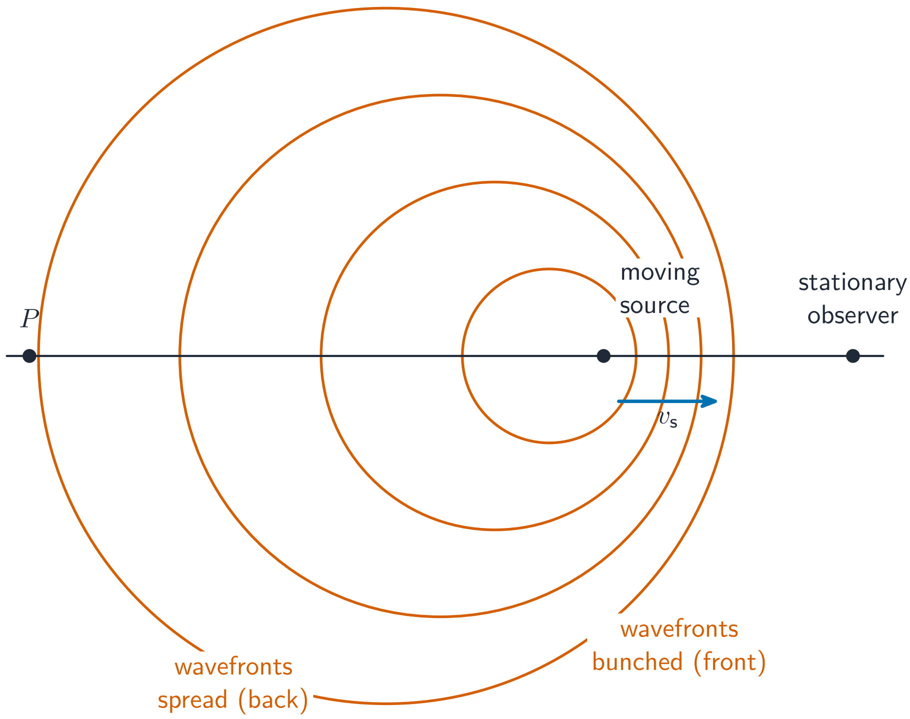


A displacement–time graph shows the wave's amplitude and period

Doppler effect (moving source, stationary observer)

When the **source** 波源 of a sound moves relative to a **stationary** 静止 **observer** 观察者, the heard frequency is different from the source frequency. This is the **Doppler effect** 多普勒效应.

- source moving towards the observer: the wavefronts in front are squashed, so the wavelength is shorter and the heard frequency is **higher**.
- source moving away from the observer: the wavefronts behind are spread out, so the wavelength is longer and the heard frequency is **lower**.



A moving source squashes the wavefronts ahead of it, raising the observed frequency

The formula (source moving at speed v_s along the line to the observer; wave speed v, source frequency f_s, heard frequency f_o):

$$f_o = \frac{v \cdot f_s}{v \pm v_s}.$$

Choose the sign to match the physics:

- minus sign on the bottom when the source moves **towards** the observer ($f_o > f_s$),
- plus sign when the source moves **away** ($f_o < f_s$).

You only need the case of a stationary observer.

Worked example. A car horn at $f_s = 800$ Hz moves at 30 m s^{-1} towards a still listener. Speed of sound $v = 340$ m s^{-1}:

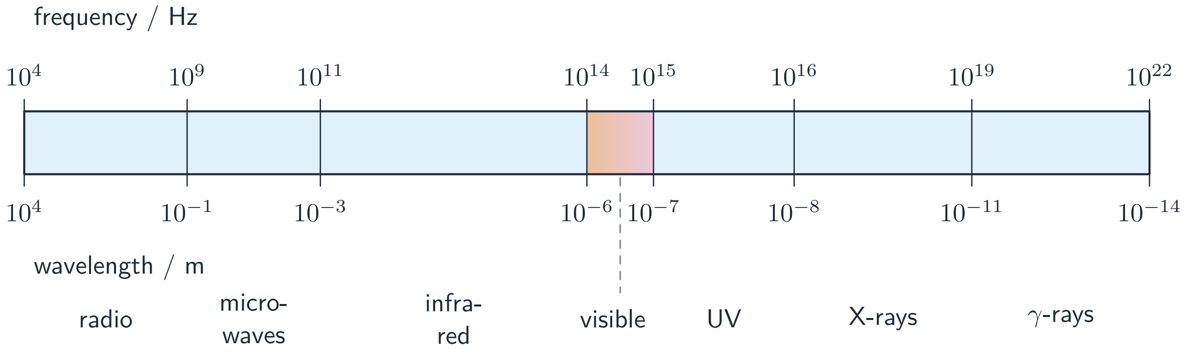
$$f_o = \frac{340 \times 800}{340 - 30} = \frac{272\,000}{310} \approx 877 \text{ Hz}.$$

Electromagnetic spectrum

All **electromagnetic waves** 电磁波 (EM waves) are **transverse** and travel in a **vacuum** 真空 at the same speed:

$$c = 3.00 \times 10^8 \text{ m s}^{-1}.$$

The **electromagnetic spectrum** 电磁波谱 includes radio waves, **microwaves** 微波, **infrared** 红外线, visible light, **ultraviolet** 紫外线, **X-rays** X 射线 and **γ-rays** 射线.



The electromagnetic spectrum

Approximate wavelength ranges in free space (learn the orders of magnitude):

- radio waves: $> 10^{-1}$ m (up to many km).
- microwaves: 10^{-3} m to 10^{-1} m.
- infrared: $\sim 7 \times 10^{-7}$ m to 10^{-3} m.
- visible light: 400 nm (violet) to 700 nm (red), i.e. 4×10^{-7} m to 7×10^{-7} m.
- ultraviolet: $\sim 10^{-8}$ m to 4×10^{-7} m.
- X-rays: $\sim 10^{-11}$ m to 10^{-8} m.
- γ-rays: $< 10^{-11}$ m.

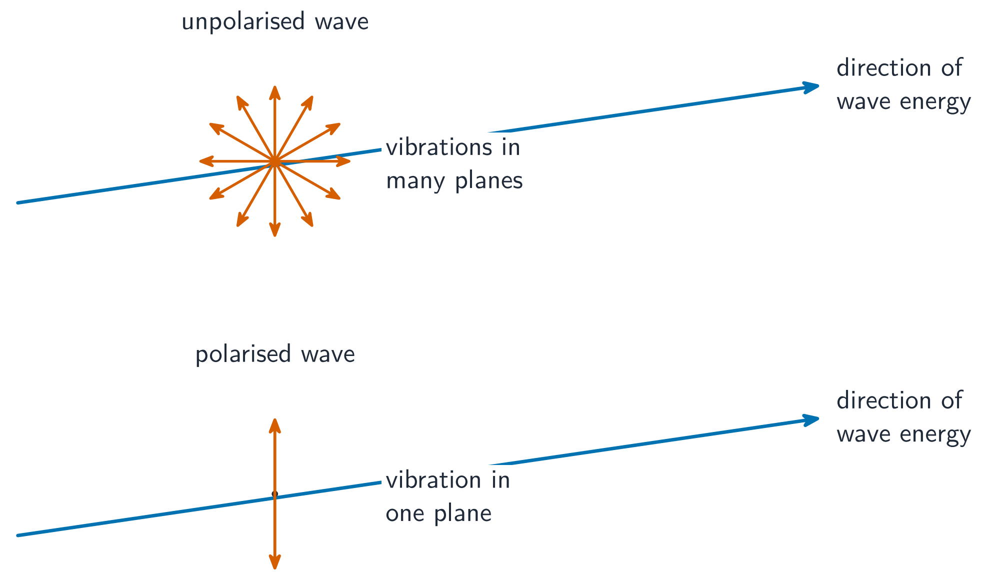
The boundaries between regions are not sharp. Use $c = f\lambda$ to change between wavelength and frequency. Only light with wavelengths 400–700 nm can be seen.

Polarisation

Polarisation 偏振 means making a transverse wave oscillate in one plane only.

- Only **transverse** waves can be polarised —the oscillation is perpendicular to the direction of travel, so different perpendicular planes are real choices.
- **Longitudinal** waves (sound) cannot be polarised —the oscillation is along the direction of travel, so there is no other plane.

So polarisation is a test: if a wave can be polarised, it must be transverse.



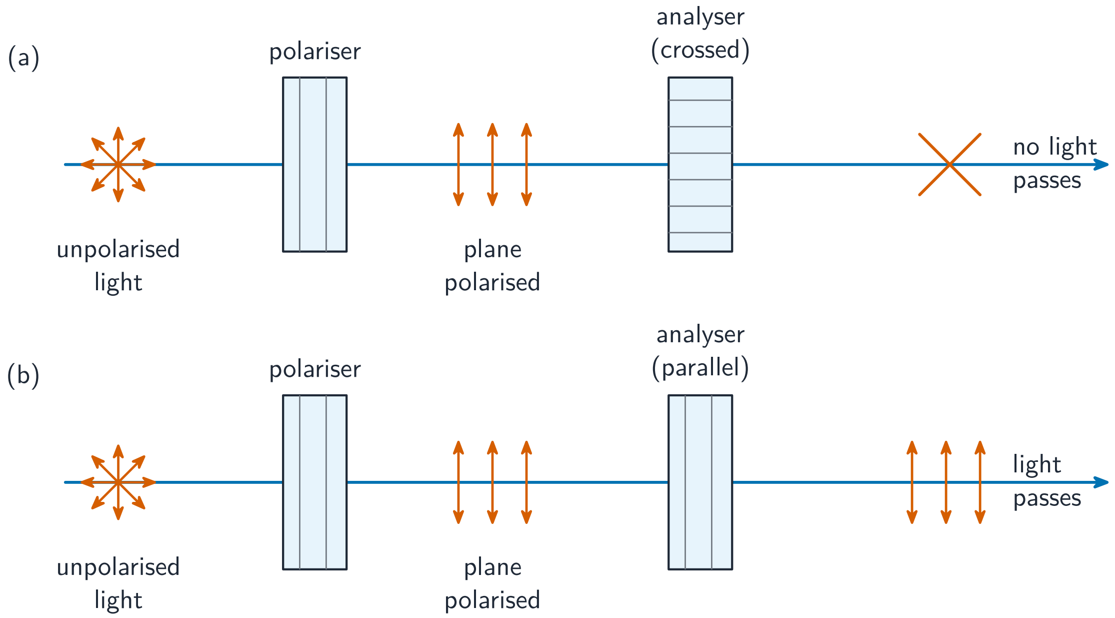
Unpolarised waves vibrate in many planes; a polarised wave vibrates in one plane

Malus's law

Plane-polarised 平面偏振 light of intensity I_0 passes through a **polarising filter** 偏振片 whose **transmission axis** 透光轴 is at angle θ to the plane of polarisation. The transmitted intensity is given by **Malus's law** 马吕斯定律:

$$I = I_0 \cos^2 \theta.$$

- $\theta = 0°$: filter lined up with the polarisation, $I = I_0$, all passes through.
- $\theta = 90°$: filter at right angles, $I = 0$, all blocked.
- $\theta = 60°$: $I = I_0 \cos^2 60° = I_0 \cdot 0.25 = I_0/4$.



Crossed filters (a) block the light; parallel filters (b) let it pass

For two filters in a row, use Malus's law twice with the angle between each pair. Be careful with the angle each time —after the first filter the polarisation is along that filter's axis, and the second filter's angle is measured from there.

(You do not need to work out the effect of a polarising filter on an unpolarised wave.)