Electric fields

A-Level Physics

Electric fields

Lightning is a giant spark driven by a huge electric field.

Image: Fir0002, GFDL 1.2 (commons.wikimedia.org)

An **electric field** 电场 is a region where a charge feels a **force** 力 from other charges. The **electric field strength** 电场强度 E at a point is the **force per unit positive charge** on a small positive **test charge** 检验电荷 placed there:

$$E = \frac{F}{q}.$$

Unit: N C^{-1} (the same as V m^{-1}, as we will see). E is a **vector** 矢量, pointing the way the force acts on a **positive** charge. The force on a charge q is

$$F = qE,$$

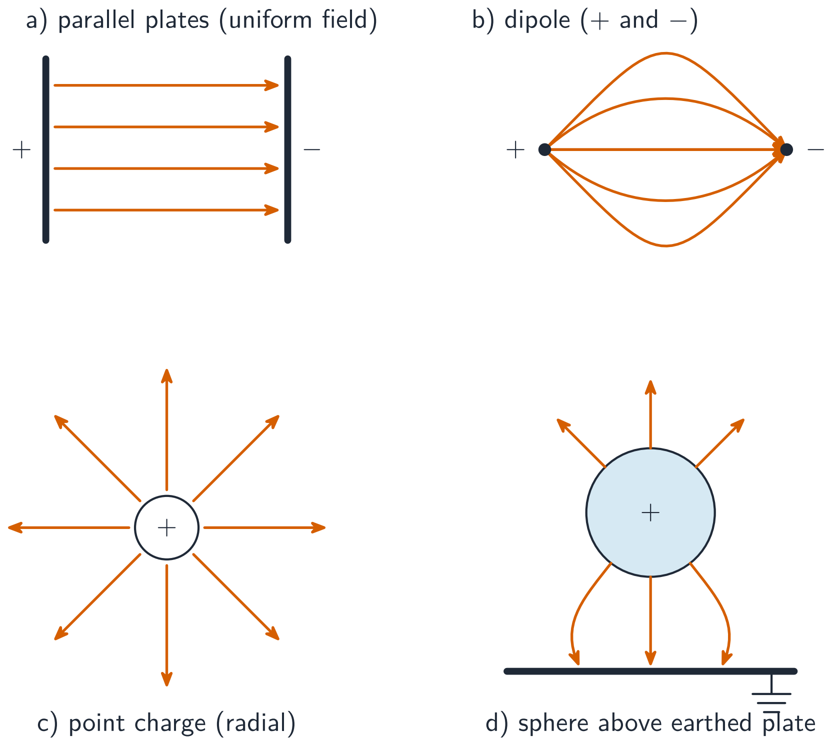
opposite to the field if q is negative.

Field lines

- **field lines** 场线 point the way the force acts on a positive test charge.
- lines start on **positive** charges and end on **negative** charges (or go to infinity).
- lines never cross; closer lines mean a stronger field.

Examples: a positive **point charge** 点电荷 has radial lines pointing out; a negative one has lines pointing in; two opposite charges (a **dipole** 偶极子) have lines curving from +

to −; two parallel charged plates give a **uniform field** 匀强场 of equally spaced parallel lines.



Field-line patterns for parallel plates, a dipole, a point charge, and a charged sphere above an earthed plate

A Van de Graaff generator stores a large static charge on its metal dome, making a strong electric field around it

Image: 3B Scientific, Product image (www.3bscientific.com)

Uniform electric fields

Between two parallel plates a distance d apart with **potential difference** 电势差 V between them, the field is uniform (apart from edge effects) with size

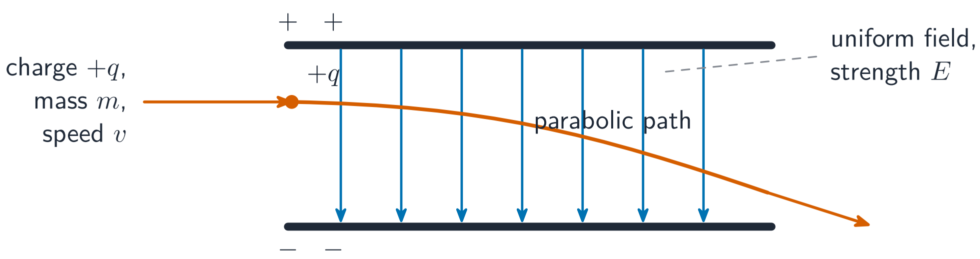
$$E = \frac{V}{d}.$$

It points from the higher-potential plate to the lower one. The unit V m^{-1} comes straight from this and equals N C^{-1}.

A charged particle in a uniform field

A charge q in a uniform field feels a **constant force** $F = qE$, so a **constant acceleration** $a = qE/m$ —just like a mass in a uniform gravitational field.

- released **at rest**, it speeds up along the field (positive charge) or against it (negative charge), gaining **kinetic energy** 动能.
- entering **at right angles** to the field, it follows a **parabolic** 抛物线 path —like a **projectile** 抛体 in gravity. This is how a **cathode-ray tube** 阴极射线管 used to steer its beam.



A charge entering a uniform field at right angles follows a parabolic path, like a projectile

Coulomb's law

For two point charges Q_1 and Q_2 a distance r apart in free space, each feels a force of size

$$F = \frac{Q_1 Q_2}{4\pi\varepsilon_0 r^2}.$$

This is **Coulomb's law** 库仑定律. Here $\varepsilon_0 = 8.85 \times 10^{-12}$ F m^{-1} is the **permittivity of free space** 真空电容率, and $1/(4\pi\varepsilon_0) \approx 9.0 \times 10^9$ N m^2 C^{-2}. The force is along the line joining the charges: **repulsive** for like charges, **attractive** for opposite charges.

Spheres treated as point charges

A spherical **conductor** 导体 with total charge Q gives, at any point **outside**, the same field as a point charge Q at its centre (measure r from the centre). **Inside** a hollow charged conductor the field is zero, so the conductor is an **equipotential** 等势面.

Electric field due to a point charge

The field at distance r from a point charge Q is

$$E = \frac{Q}{4\pi\varepsilon_0 r^2}.$$

It points out from a positive Q, in towards a negative Q, and falls as $1/r^2$ —just like **gravitational field** 重力场 strength, except gravity is always attractive. For several charges, add the fields as a **vector sum** 矢量和.

Electric potential

Electric potential 电势 V at a point is the **work done per unit positive charge** in bringing a small positive test charge from **infinity** 无穷远 to that point:

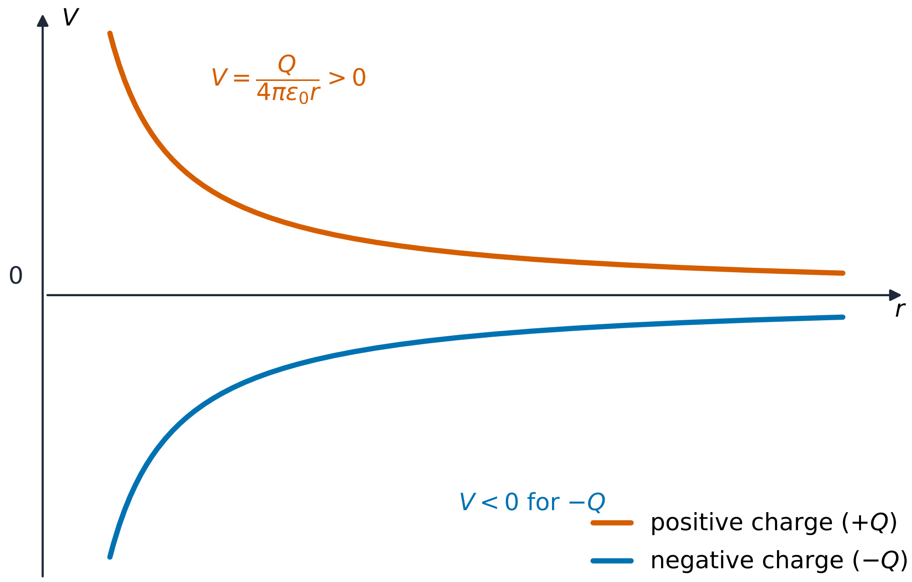
$$V = \frac{W}{q}.$$

Unit: V. The potential is **zero at infinity**. For a positive source charge $V > 0$ everywhere outside; for a negative source charge $V < 0$.

Potential due to a point charge

$$V = \frac{Q}{4\pi\varepsilon_0 r}.$$

Note the $1/r$ here (compared with $1/r^2$ for the field). V is a **scalar** 标量; for several charges, add the potentials (with sign).



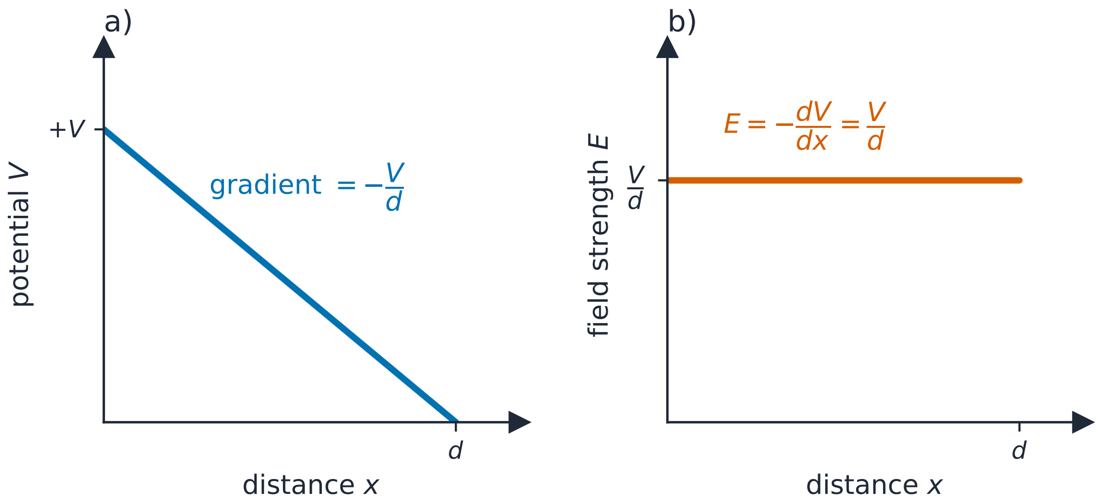
The potential near a point charge varies as $1/r$ —positive for a positive charge, negative for a negative one

Link between field and potential

The field equals the **negative potential gradient** 电势梯度:

$$E = -\frac{dV}{dx}.$$

Between parallel plates V changes evenly with position, giving $E = V/d$ as before. The minus sign means the field points towards **lower** potential. For a point charge, $-\frac{dV}{dr} = \frac{Q}{4\pi\varepsilon_0 r^2} = E$.



In a uniform field the potential falls steadily with distance, so the field strength V/d is constant

Electric potential energy

A charge q at a point of potential V has **electric potential energy** 电势能 $E_P = qV$. For two point charges Q and q a distance r apart:

$$E_P = \frac{Qq}{4\pi\varepsilon_0 r}.$$

- **like** charges: $E_P > 0$ —stored energy that would be released if they flew apart.
- **opposite** charges: $E_P < 0$ —a **bound** 束缚 system; energy must be supplied to separate them.

In both cases $E_P \to 0$ as $r \to \infty$.

Worked-example pattern

An **electron** 电子 orbits a **nucleus** 原子核 of charge $+Ze$ at distance r. The Coulomb attraction provides the **centripetal force** 向心力:

$$\frac{Ze^2}{4\pi\varepsilon_0 r^2} = \frac{m_e v^2}{r}, \qquad v = \sqrt{\frac{Ze^2}{4\pi\varepsilon_0 m_e r}}.$$

The total energy is kinetic plus potential:

$$E_{\text{total}} = \tfrac{1}{2}m_e v^2 - \frac{Ze^2}{4\pi\varepsilon_0 r} = -\frac{Ze^2}{8\pi\varepsilon_0 r},$$

which is negative (a bound state).

Gravitational versus electric

The two field theories look alike:

Quantity	**Gravitational**	**Electric**
Source	mass M (always positive)	charge Q (can be $\pm$)
Field strength	$g = GM/r^2$	$E = Q/(4\pi\varepsilon_0 r^2)$
Force on test object	$F = mg$	$F = qE$
Potential	$\phi = -GM/r$	$V = Q/(4\pi\varepsilon_0 r)$
PE of two	$-GMm/r$	$Qq/(4\pi\varepsilon_0 r)$
Nature	always attractive	attractive or repulsive

The minus sign in the **gravitational potential** 引力势 reflects that gravity is always attractive; the electric potential takes the sign of the source charge.