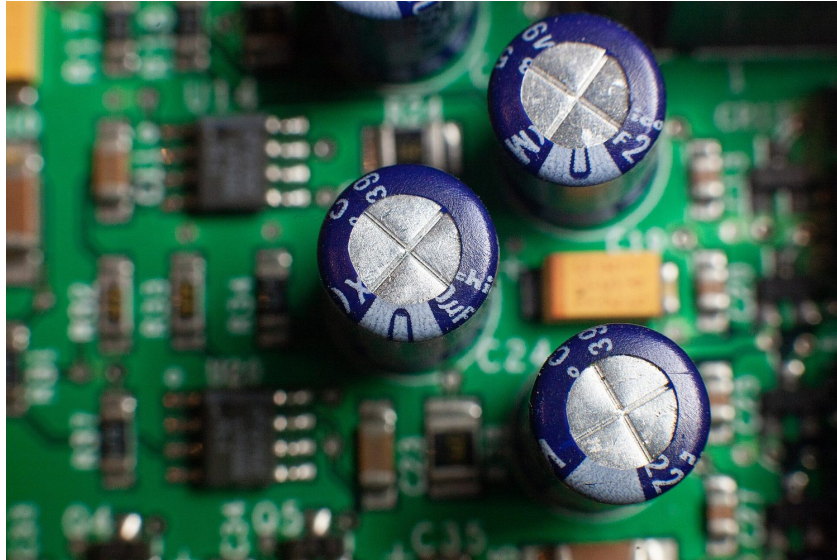


Capacitance

A-Level Physics

Capacitance



Capacitors store electric charge in a circuit.

Image: Giovanna 27, CC BY 4.0 (commons.wikimedia.org)

A **capacitor** 电容器 stores charge. The simplest one is two parallel **conductor** 导体 plates with an **insulator** 绝缘体 (a **dielectric** 电介质, or just **vacuum** 真空 / air) between them. Connected to a battery, charge $+Q$ builds up on one plate and $-Q$ on the other, with a **potential difference** 电势差 V across the gap.

The **capacitance** 电容 C of any capacitor (or any isolated conductor) is

$$C = \frac{Q}{V}.$$

This applies to:

- an **isolated sphere** holding charge Q at potential V (zero at infinity). For radius r , $V = Q/(4\pi\epsilon_0 r)$, so $C = 4\pi\epsilon_0 r$.
- a **parallel-plate capacitor**: charges $\pm Q$ on the plates, p.d. V between them.

Unit: **farad** 法拉 (F) = C V⁻¹. A farad is huge, so real capacitors run from pF to mF.

Capacitance is **constant** for a given capacitor (set by its size and dielectric). Doubling the charge doubles the voltage, so $C = Q/V$ stays the same.



Real capacitors range from large electrolytic cans (high capacitance) down to tiny film and ceramic types – from pF up to mF

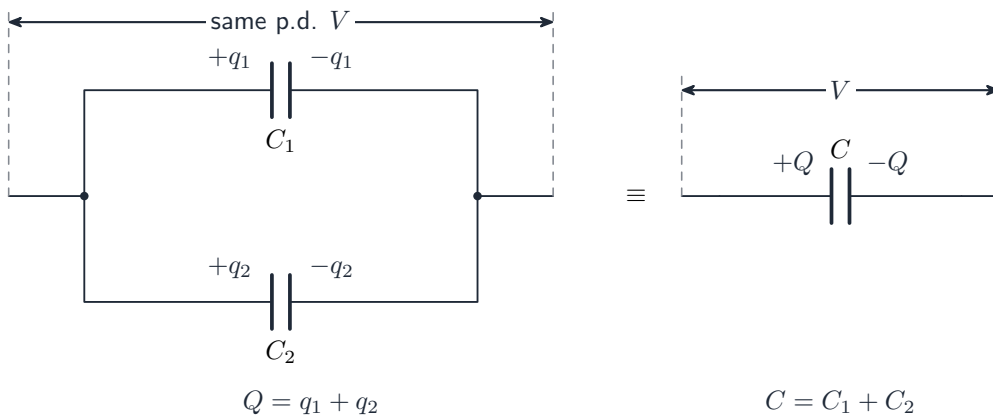
Image: Intiti, Public domain (commons.wikimedia.org)

Combining capacitors

Capacitors in parallel share the same p.d. V . The total charge is the sum:

$$Q_{\text{total}} = C_1V + C_2V + \dots, \quad \text{so} \quad C_{\text{parallel}} = C_1 + C_2 + \dots$$

A parallel combination has **larger** capacitance than any one capacitor.

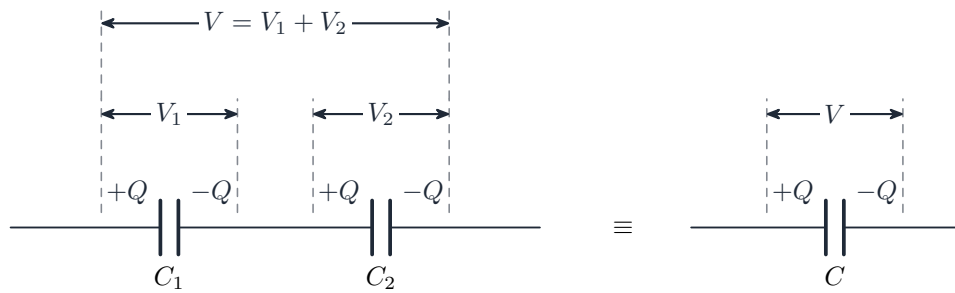


Capacitors in parallel share the same p.d.; the charges add

Capacitors in series carry the **same charge** Q . The total p.d. is the sum:

$$V_{\text{total}} = \frac{Q}{C_1} + \frac{Q}{C_2} + \dots, \quad \text{so} \quad \frac{1}{C_{\text{series}}} = \frac{1}{C_1} + \frac{1}{C_2} + \dots$$

A series combination has **smaller** capacitance than any one capacitor.



Capacitors in series carry the same charge; the p.d.s add

Note: these rules are the **opposite** of those for resistors (resistors sum in series; capacitors sum in parallel), because $C = Q/V$ has V on the bottom while $R = V/I$ has I on the bottom.

Energy stored in a capacitor

Charging a capacitor from 0 to Q needs work, because each extra bit of charge is pushed against the p.d. already there. When the charge is q , the p.d. is $V(q) = q/C$, so adding a small charge dq needs work $V dq$. The total work is

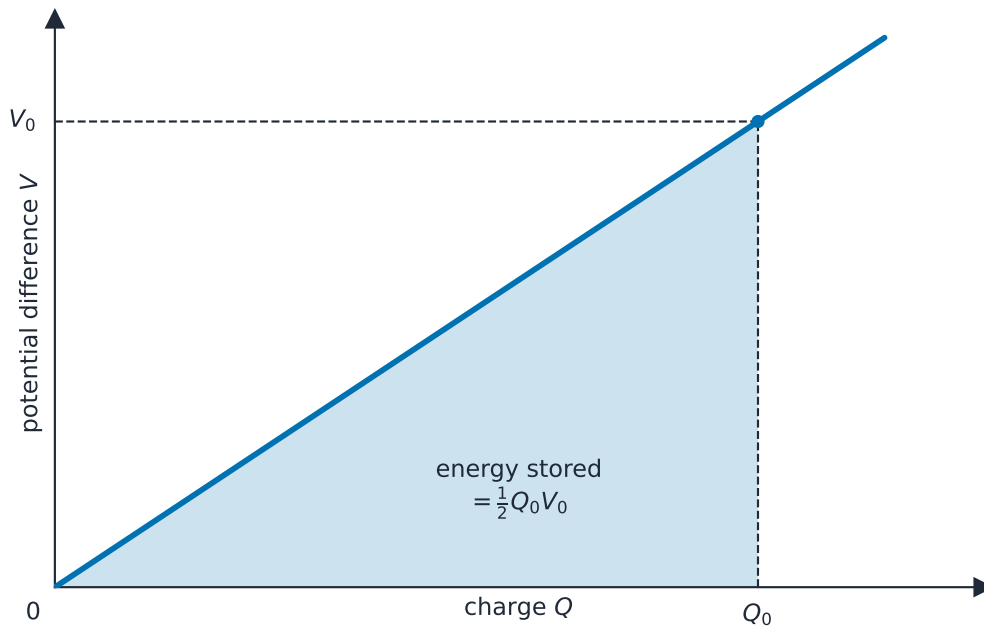
$$W = \int_0^Q \frac{q}{C} dq = \frac{Q^2}{2C}.$$

Using $V = Q/C$, this is the **energy** 能量 stored:

$$W = \frac{1}{2}QV = \frac{1}{2}CV^2 = \frac{Q^2}{2C}.$$

Reading the Q - V graph

A plot of V against Q is a straight line through the origin with gradient $1/C$. The energy stored is the **area** under the line up to a given charge Q , which is the triangle $\frac{1}{2}QV$. The factor $\frac{1}{2}$ is there because the average p.d. during charging is $V/2$ (it grows from zero to V), not V .



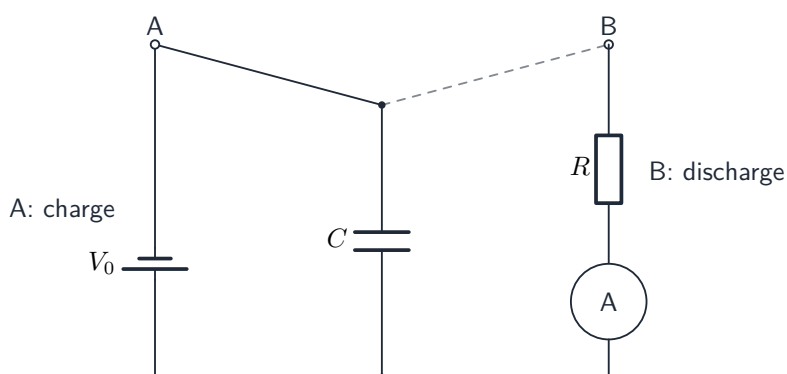
The energy stored is the area under the potential-charge line (the triangle $\frac{1}{2}QV$)

Why charging is "half efficient"

Connect a capacitor C to an ideal battery of e.m.f. V through a wire. The capacitor stores $\frac{1}{2}CV^2$, but the battery supplies charge $Q = CV$ at e.m.f. V , giving out $QV = CV^2$. The other half is lost as heat in the wire —whatever the wire's resistance.

Capacitor discharging through a resistor

A capacitor C charged to V_0 is connected through a switch to a **resistor** 电阻器 of **resistance** 电阻 R . When the switch closes at $t = 0$, the capacitor discharges.



A capacitor charges through switch A, then discharges through the resistor via switch B

Setting up the equation

By **Kirchhoff's second law** 基尔霍夫第二定律 around the loop, $V_C = V_R$. Using $V_C = Q/C$, $V_R = IR$ and $I = -dQ/dt$:

$$\frac{Q}{C} = -R \frac{dQ}{dt}.$$

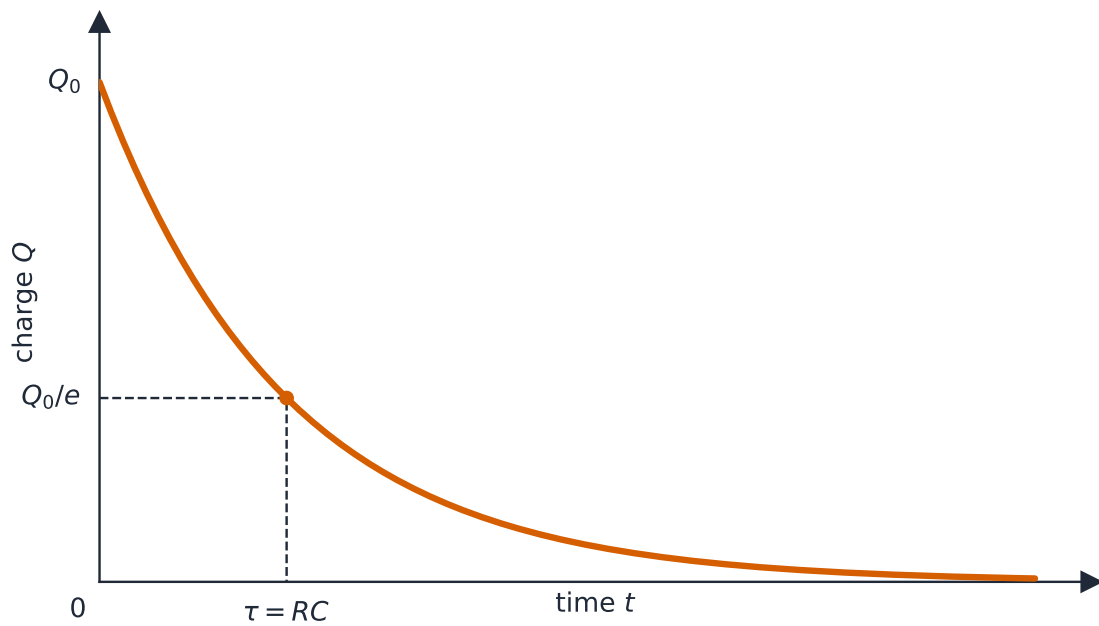
This is solved by an **exponential decay** 指数衰减 with time constant RC .

Discharge equations

Charge Q , p.d. V and current I all decay exponentially with the same time constant:

$$Q = Q_0 e^{-t/(RC)}, \quad V = V_0 e^{-t/(RC)}, \quad I = I_0 e^{-t/(RC)},$$

with $I_0 = V_0/R$.



Charge decays exponentially during discharge, falling to Q_0/e after one time constant RC

Time constant

$$\tau = RC$$

is the **time constant** 时间常数 (in seconds: $\Omega \cdot \text{F} = \text{s}$). It is the time for a decaying quantity to fall to $1/e \approx 0.37$ (about 37%) of its starting value. After 2τ it is at about 13.5%; after 5τ , below 1%.

To find τ from a curve: read the time to fall to $1/e$ of the start. Or take logs: $\ln(V/V_0) = -t/(RC)$, so a plot of $\ln V$ against t is a straight line with gradient $-1/(RC)$.

Reading graphs during discharge

- Q against V_C : since $Q = CV$ always, this is a straight line through the origin with gradient C . Discharge moves the point from (V_0, Q_0) down to $(0, 0)$.

- I against V_C : since $I = V_C/R$, this is a straight line through the origin with gradient $1/R$, so you can find R .

Common exam questions

Given a discharge curve $V(t)$ or $Q(t)$:

- read the start value V_0 or Q_0 at $t = 0$.
- read the time to fall to $V_0/e \rightarrow$ time constant $\tau = RC$.
- given R , find $C = \tau/R$ (or the other way round).
- predict a later value with the exponential formula.