Medical physics

A-Level Physics

Ultrasound

Piezo-electric effect

A **piezo-electric** 压电 crystal changes shape a little when a p.d. is put across it, and the reverse: it makes an **electromotive force** 电动势 (e.m.f.) across itself when its shape is changed. Quartz and PZT are common examples. Two linked effects:

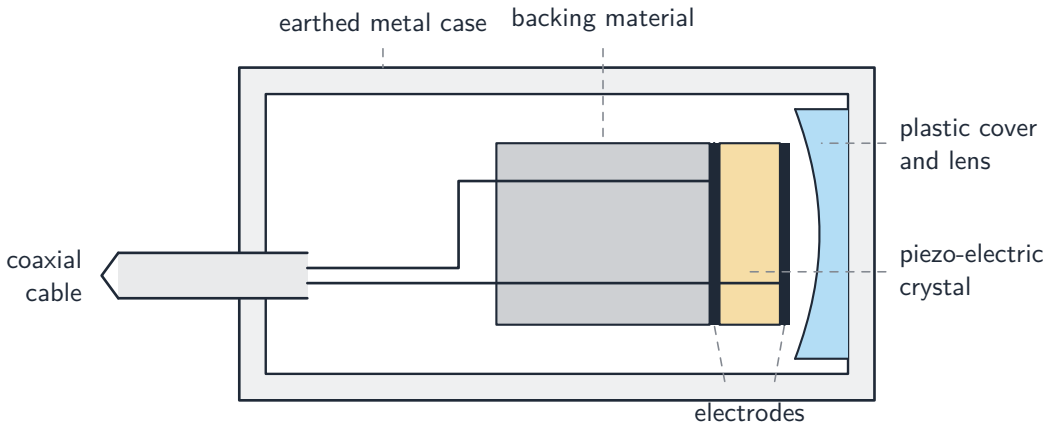
- **apply a p.d.** → the crystal changes shape (used to make vibrations).
- **change the shape** (a wave squeezes it) → an e.m.f. appears (used to detect vibrations).

Piezo-electric transducer

A **transducer** 换能器 uses this effect to both **make** and **detect ultrasound** 超声波.

- an alternating p.d. (a few MHz) makes the crystal vibrate at the same frequency, sending out **longitudinal** 纵波 waves above 20 kHz (1–10 MHz for medical imaging).
- the same crystal then detects: returning ultrasound makes it vibrate and produce an e.m.f.

So one transducer is both emitter and detector, switching between sending and listening.



A piezo-electric transducer both sends and detects ultrasound using a vibrating crystal

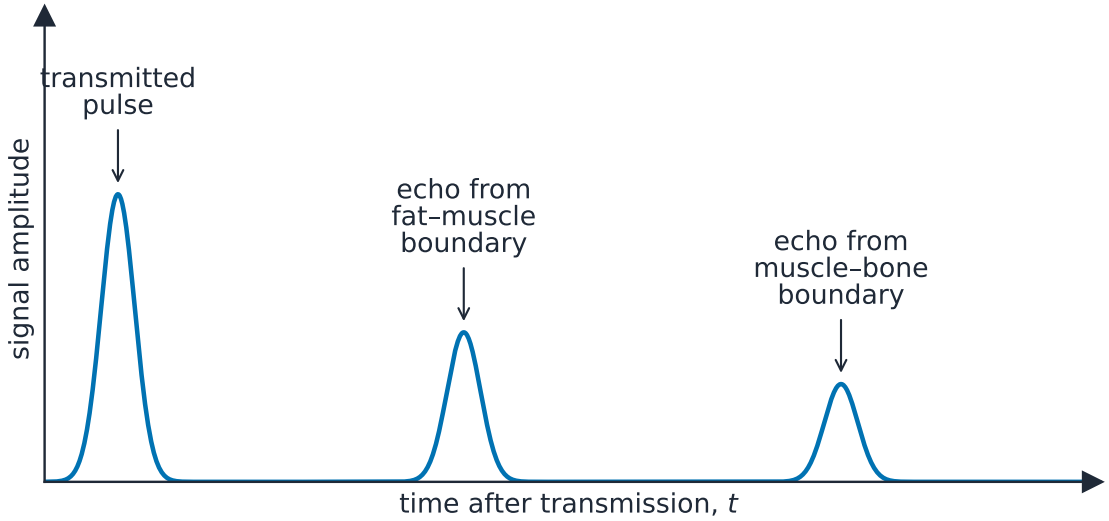
Pulse-echo imaging

To see inside the body (**pulse-echo** 脉冲回波 imaging):

1. the transducer sends a **short pulse** into the body.
2. at each tissue boundary, part of the pulse is **reflected** and part goes on.
3. the transducer detects each reflected pulse.

4. the **time delay** gives the depth: $d = ct/2$ (there and back). The echo's amplitude gives the strength of the reflection.
5. sweeping across the body builds a 2-D image.

A **coupling gel** 耦合剂 is put between the transducer and the skin to push out the air; without it almost all the ultrasound would reflect at the skin–air boundary and never enter the body.



An A-scan shows the transmitted pulse and the echoes from each tissue boundary

Specific acoustic impedance

The **specific acoustic impedance** 声阻抗 of a medium is

$$Z = \rho c,$$

where ρ is the **density** 密度 and c the speed of sound. Unit: kg m^{-2} s^{-1}. Bone has large Z; air has small Z; soft tissue is in between.

Reflection at a boundary

At a boundary between media of impedance Z_1 and Z_2, the **intensity reflection coefficient** 强度反射系数 (fraction reflected) is

$$\frac{I_\mathrm{R}}{I_0} = \left(\frac{Z_1 - Z_2}{Z_1 + Z_2}\right)^2.$$

- very different impedances: almost all is reflected (skin/air —hence the gel).
- very similar impedances: almost nothing is reflected, so the boundary cannot be seen.
- best for imaging: different enough to give an echo, but not so different that nothing passes on.

Attenuation

As ultrasound goes through tissue, its intensity falls with distance:

$$I = I_0 e^{-\mu x},$$

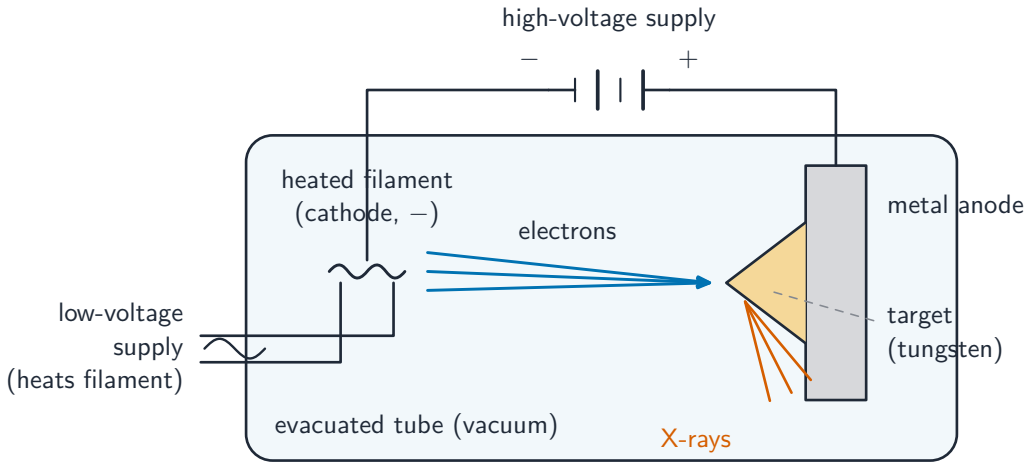
where μ is the **attenuation coefficient** 衰减系数 (unit m^{-1}). The same form applies to X-rays.

X-rays

Production

X-rays come from an **X-ray tube** X 射线管:

1. a heated **cathode** 阴极 emits electrons by **thermionic emission** 热电子发射.
2. a high p.d. (tens to hundreds of kV) accelerates the electrons across a **vacuum** 真空 to a metal **target** 靶 (the **anode** 阳极, often **tungsten** 钨).
3. the electrons hit the target and slow sharply. Most of their **kinetic energy** 动能 becomes heat; a small part is emitted as X-ray **photons** 光子 (**Bremsstrahlung** 轫致辐射, "braking radiation"). Some electrons knock out inner electrons of the metal atoms, and the refilling emits **characteristic** 特征 X-ray lines.



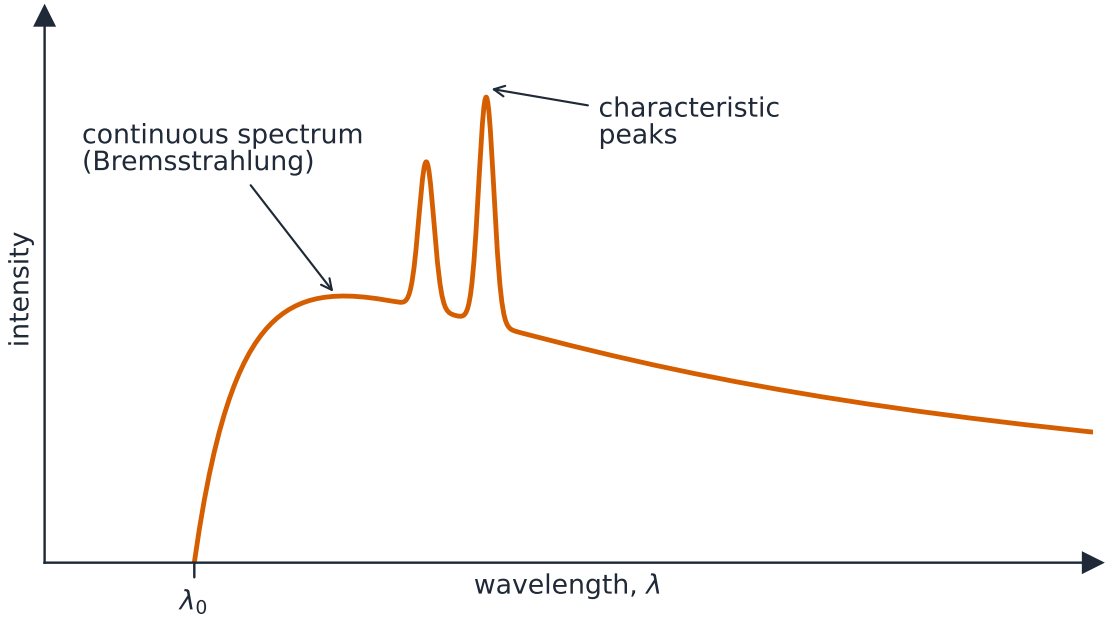
In an X-ray tube electrons from the heated cathode are accelerated onto a metal target anode

Minimum wavelength

The most energy one X-ray photon can have is the full kinetic energy of one accelerated electron, lost in a single event. For accelerating p.d. V, the KE is eV, so

$$hf_{\max} = eV, \qquad \lambda_{\min} = \frac{hc}{eV}.$$

This is the short-wavelength cut-off. The continuous Bremsstrahlung spectrum tails off above $\lambda_{\min}$, with sharp characteristic peaks set by the target metal.

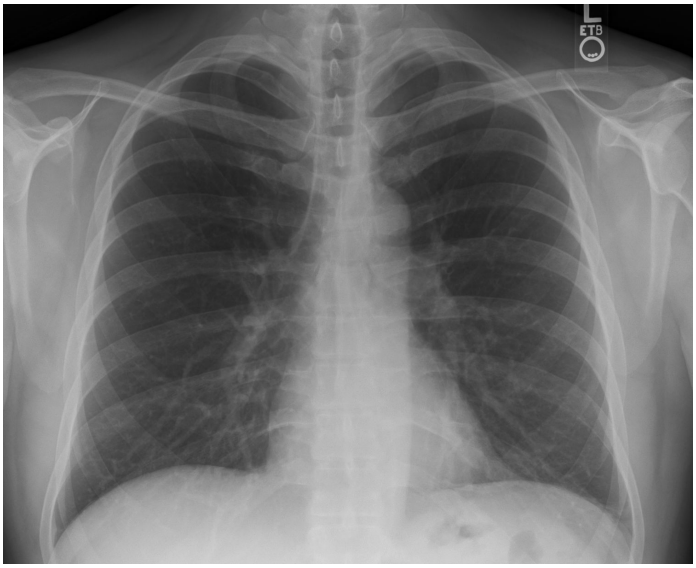


A typical X-ray spectrum — a continuous Bremsstrahlung curve cut off at λ_0, with sharp characteristic peaks

Imaging with X-rays

X-rays pass through the patient onto a detector. Tissues that **attenuate** 衰减 more (bone, high Z) cast a stronger shadow and look lighter; tissues that attenuate less (soft tissue, lung) look darker.

The **contrast** 对比度 is the difference in attenuation between tissues. A **contrast medium** 造影剂 (e.g. a barium meal) can be given to make soft tissues stand out.



A real chest X-ray: dense bone absorbs more X-rays and looks white; the air-filled lungs let X-rays through and look dark

Image: Stillwaterising, CC0 (commons.wikimedia.org)

Attenuation law

$$I = I_0 e^{-\mu x}.$$

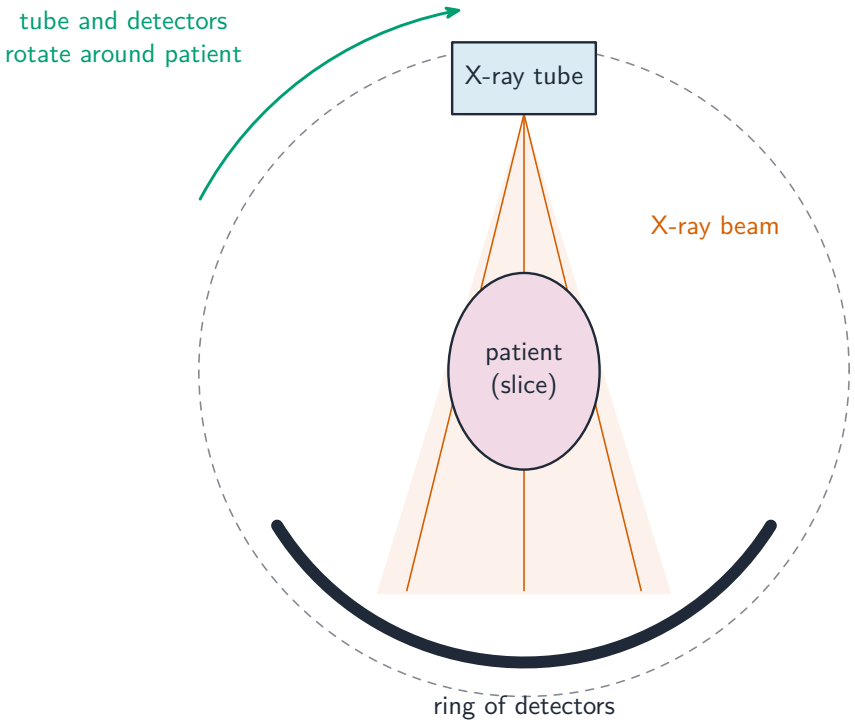
Higher-energy X-rays penetrate further (smaller μ); bone has a much larger μ than soft tissue. To find the thickness for a given fraction, take logs: $x = \dfrac{1}{\mu} \ln \dfrac{I_0}{I}$. The **half-value thickness** 半值厚度 $x_{1/2} = \ln 2/\mu$ halves the intensity (like half-life in decay).

Computed tomography (CT)

A **computed tomography** 计算机断层扫描 (CT) scan builds a 3-D image:

1. the tube and detectors rotate around the patient, taking many images of one thin slice from **different angles**.
2. a computer combines these into a 2-D cross-section of the slice.
3. the patient is moved along, and the next slice is imaged.
4. the slices are stacked into a 3-D image.

CT shows far more than a single X-ray, because overlapping soft tissues are separated by the many-angle reconstruction.



In a CT scan the X-ray tube and detectors rotate around the patient to image a slice from many angles

PET scanning

Tracer

A **tracer** 示踪剂 is a substance with radioactive nuclei put into the body. It is taken up more by the tissue being studied (e.g. a tumour takes up more glucose-tagged tracer due to its high **metabolism** 代谢). Its decay is detected from outside.

In **positron emission tomography** 正电子发射断层扫描 (PET), the tracer is a β^+ emitter —it gives out a **positron** 正电子. A common one is fluorine-18 on a glucose analogue (FDG).

Annihilation

When a particle meets its **antiparticle** 反粒子 they **annihilate** 湮灭: their mass turns into electromagnetic energy. In PET:

- a positron travels a few mm before meeting an **electron** 电子.
- they annihilate. Energy and **momentum** 动量 are conserved.
- since the total momentum is about zero, two photons are produced going in **opposite directions**, each 511 keV ($= m_e c^2$).

Energy of the annihilation photons

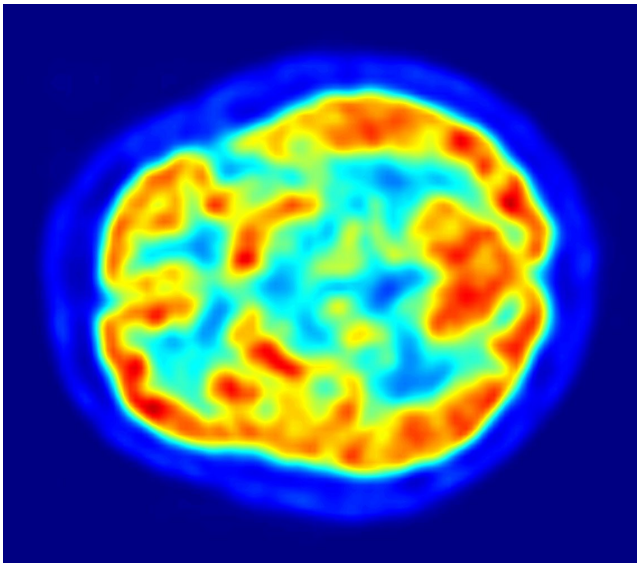
By energy conservation, the total photon energy equals the pair's rest **energy** 能量:

$$2hf = 2m_e c^2, \qquad hf = m_e c^2.$$

Each photon has $hf = m_e c^2 \approx 8.2 \times 10^{-14}$ J ≈ 0.51 MeV, with $\lambda \approx 2.4 \times 10^{-12}$ m.

Reconstructing the image

The two photons leave the body in opposite directions and hit **detector rings** around the patient. Recording the **two simultaneous arrivals** (a "coincidence") fixes the line the annihilation happened on. Many coincidences from many angles let the computer build a 3-D map of the tracer —showing tissues with high metabolic activity. Comparing the two **arrival times** can refine the position along that line (time-of-flight PET).



A finished PET image of the brain: warm colours (red, yellow) mark where the tracer collected – the most active tissue

Image: Jens Maus (http://jens-maus.de/), Public domain (commons.wikimedia.org)