

## 2.4 When Does a Derivative Exist?

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 9 marks

### Objective

Build the skills to answer exam questions on **differentiability and continuity**.

You must be able to:

- use that **differentiability** 可导性 implies **continuity** 连续性 (but not the reverse)
- identify where a continuous function fails to be differentiable

### 1 Worked examples

Study these first. Each one shows the method for a question type used later.

#### ■ Differentiability and continuity

If  $f$  is differentiable at  $a$ , then  $f$  is continuous at  $a$ . The converse **fails**:  $f$  can be continuous yet not differentiable—at a **corner**, cusp, or vertical tangent.

#### ■ Example

$f(x) = |x|$  is continuous at 0 but has a corner there, so it is not differentiable at 0.

### 2 Practice

2.1 State the implication between differentiability and continuity. [1]

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2.2 State whether continuity implies differentiability. [1]

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2.3 Give an example of a continuous but non-differentiable point. [2]

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### 3 Exam-style questions

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**3.1** If  $f$  is differentiable at  $a$ , then  $f$  is [1]

- **A** discontinuous
  - **B** continuous at  $a$
  - **C** linear
  - **D** constant
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**3.2**  $f(x) = |x|$  at  $x = 0$  is [1]

- **A** differentiable
  - **B** continuous but not differentiable
  - **C** discontinuous
  - **D** undefined
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**3.3** Continuous functions can fail to be differentiable at certain features.

(a) Name the feature at a sharp point. [1]

(b) Name the feature with an infinite slope. [1]

(c) State whether  $|x|$  is differentiable at 0. [1]

## 4 Go further

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- work through the **2.4 Connecting Differentiability and Continuity** lesson on the **Learn** page;
- read the **Differentiation: Definition and Fundamental Properties** section of the AP Calculus AB handout on the **Know** page.

## Solutions

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**2.1** differentiable at  $a \Rightarrow$  continuous at  $a$ .

**2.2** no (continuity does not imply differentiability).

**2.3**  $f(x) = |x|$  at  $x = 0$  (a corner).

**3.1** B.

**3.2** B.

**3.3** (a) a corner. (b) a vertical tangent. (c) no.