

2.3 Estimating a Derivative at a Point

Name: _____ Class: _____ Date: _____

Total: 9 marks

Objective

Build the skills to answer exam questions on **estimating derivatives at a point**.

You must be able to:

- estimate $f'(a)$ from a **table** 表格 of values using a small- h difference quotient

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Estimating a derivative

Estimate $f'(a)$ with a small- h difference quotient from table values:

$$f'(a) \approx \frac{f(a+h) - f(a)}{h} \quad \text{or} \quad \frac{f(a+h) - f(a-h)}{2h} \quad (\text{symmetric}).$$

■ Example

$$f(2) = 5, f(2.1) = 5.4: f'(2) \approx \frac{5.4 - 5}{0.1} = 4.$$

2 Practice

2.1 State a formula to estimate $f'(a)$ from a table. [1]

2.2 If $f(2) = 5$ and $f(2.1) = 5.4$, estimate $f'(2)$. [2]

2.3 State why a smaller h generally gives a better estimate. [1]

3 Exam-style questions

3.1 A derivative at a point can be estimated from a table using a [1]

- **A** single value
 - **B** difference quotient with small h
 - **C** maximum
 - **D** mean
-

3.2 The symmetric difference quotient is [1]

- **A** $\frac{f(a+h) - f(a)}{h}$
 - **B** $\frac{f(a+h) - f(a-h)}{2h}$
 - **C** $\frac{f(a)}{h}$
 - **D** $f(a+h) \cdot f(a-h)$
-

3.3 $f(3) = 10$, $f(3.2) = 10.6$.

(a) Find Δf . [1]

(b) Find Δx . [1]

(c) Estimate $f'(3)$. [1]

4 Go further

- work through the **2.3 Estimating Derivatives of a Function at a Point** lesson on the **Learn** page;
- read the **Differentiation: Definition and Fundamental Properties** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $f'(a) \approx \frac{f(a+h) - f(a)}{h}$ for small h .

2.2 $\frac{5.4 - 5}{2.1 - 2} = \frac{0.4}{0.1} = 4$.

2.3 a smaller interval is closer to the tangent slope (the limit).

3.1 B.

3.2 B.

3.3 (a) 0.6. (b) 0.2. (c) $\frac{0.6}{0.2} = 3$.