

2.4 Differentiability and Continuity: When a Derivative Exists

Name: _____ Class: _____ Date: _____

Total: 9 marks

Objective

Build the skills to answer exam questions on **differentiability and continuity**.

You must be able to:

- use that **differentiability** 可导性 implies **continuity** 连续性 (but not the reverse)
- identify where a continuous function fails to be differentiable

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Differentiability and continuity

If f is differentiable at a , then f is continuous at a . The converse **fails**: f can be continuous yet not differentiable—at a **corner**, cusp, or vertical tangent.

■ Example

$f(x) = |x|$ is continuous at 0 but has a corner there, so it is not differentiable at 0.

2 Practice

2.1 State the implication between differentiability and continuity. [1]

2.2 State whether continuity implies differentiability. [1]

2.3 Give an example of a continuous but non-differentiable point. [2]

3 Exam-style questions

3.1 If f is differentiable at a , then f is [1]

- **A** discontinuous
 - **B** continuous at a
 - **C** linear
 - **D** constant
-

3.2 $f(x) = |x|$ at $x = 0$ is [1]

- **A** differentiable
 - **B** continuous but not differentiable
 - **C** discontinuous
 - **D** undefined
-

3.3 Continuous functions can fail to be differentiable at certain features.

- (a) Name the feature at a sharp point. [1]
- (b) Name the feature with an infinite slope. [1]
- (c) State whether $|x|$ is differentiable at 0. [1]

4 Go further

- work through the **2.4 Connecting Differentiability and Continuity** lesson on the **Learn** page;
- read the **Differentiation: Definition and Fundamental Properties** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 differentiable at $a \Rightarrow$ continuous at a .

2.2 no (continuity does not imply differentiability).

2.3 $f(x) = |x|$ at $x = 0$ (a corner).

3.1 B.

3.2 B.

3.3 (a) a corner. (b) a vertical tangent. (c) no.