

# 6.12 Integrating Using Linear Partial Fractions

Name: \_\_\_\_\_ Class: \_\_\_\_\_ Date: \_\_\_\_\_

Total: 16 marks

## Objective

Build the skills to answer exam questions on **integration by linear partial fractions**—splitting a rational function to integrate it.

**You must be able to:**

- factor the denominator and write the **partial-fraction** 部分分式 decomposition
- solve for the unknown constants
- integrate each simple fraction to a **logarithm**

## 1 Worked examples

Study these first. Each one shows the method for a question type used later—follow the steps and you can do the Practice and Exam-style questions yourself.

### ■ The decomposition

A proper fraction with a factored denominator splits into simpler pieces:

$$\frac{1}{(x-1)(x+2)} = \frac{A}{x-1} + \frac{B}{x+2}.$$

### ■ Solving for A and B

Multiply through by  $(x-1)(x+2)$ :  $1 = A(x+2) + B(x-1)$ . Set  $x = 1$ :  $1 = 3A \Rightarrow A = \frac{1}{3}$ . Set  $x = -2$ :  $1 = -3B \Rightarrow B = -\frac{1}{3}$ .

### ■ Integrating the pieces

Each piece integrates to a log:

$$\int \left( \frac{1/3}{x-1} - \frac{1/3}{x+2} \right) dx = \frac{1}{3} \ln |x-1| - \frac{1}{3} \ln |x+2| + C.$$

### ■ When to divide first

If the fraction is **improper** (numerator degree  $\geq$  denominator), do **long division** first, then apply partial fractions to the remainder.

## 2 Practice

---

Now apply the methods above.

**2.1** Write the partial-fraction form of  $\frac{1}{(x-3)(x+1)}$  (with unknowns  $A, B$ ). [1]

---

**2.2** Find  $A$  and  $B$  for  $\frac{5}{(x-2)(x+3)} = \frac{A}{x-2} + \frac{B}{x+3}$ . [3]

**2.3** Integrate  $\int \left( \frac{2}{x-1} + \frac{3}{x+4} \right) dx$ . [2]

## 3 Exam-style questions

---

**3.1** After decomposing,  $\int \frac{A}{x-a} dx =$  [1]

- **A**  $A \ln|x-a| + C$
  - **B**  $\frac{A}{(x-a)^2} + C$
  - **C**  $A(x-a) + C$
  - **D**  $\frac{A}{2}(x-a)^2 + C$
- 

**3.2** Decompose and integrate  $\int \frac{1}{(x-1)(x+1)} dx$ .

(a) Find the constants. [2]

(b) Complete the integration.

[2]

**3.3** Find  $\int \frac{x+7}{(x-1)(x+3)} dx$ .

[5]

## 4 Go further

---

You are now ready for the real exam questions on this subtopic:

- work through the **6.12 Integrating Using Linear Partial Fractions** lesson on the **Learn** page;
- read the **Integrating Using Linear Partial Fractions** section of the AP Calculus BC handout on the **Know** page.

## Solutions

---

**2.1**  $\frac{A}{x-3} + \frac{B}{x+1}.$

**2.2**  $5 = A(x+3) + B(x-2); x=2: 5 = 5A \Rightarrow A=1; x=-3: 5 = -5B \Rightarrow B=-1.$

**2.3**  $2\ln|x-1| + 3\ln|x+4| + C.$

**3.1** **A**  $-\int \frac{A}{x-a} dx = A \ln|x-a| + C.$

**3.2** (a)  $1 = A(x+1) + B(x-1); x=1: A = \frac{1}{2}; x=-1: B = -\frac{1}{2}.$  (b)  $\frac{1}{2}\ln|x-1| - \frac{1}{2}\ln|x+1| + C.$

**3.3**  $x+7 = A(x+3) + B(x-1); x=1: 8 = 4A \Rightarrow A=2; x=-3: 4 = -4B \Rightarrow B=-1;$   
 $\int \left(\frac{2}{x-1} - \frac{1}{x+3}\right) dx = 2\ln|x-1| - \ln|x+3| + C.$