

8.9 Volume with Disc Method: Revolving Around the x- or y-Axis

Name: _____ Class: _____ Date: _____

Total: 16 marks

Objective

Build the skills to answer exam questions on the **disc method** —volumes of revolution with no hole.

You must be able to:

- revolve a region around an axis and integrate $\pi(\text{radius})^2$ (the **disc method** 圆盘法)
- set the radius as the distance from the curve to the axis
- integrate in x (around the x -axis) or y (around the y -axis)

1 Worked examples

Study these first. Each one shows the method for a question type used later —follow the steps and you can do the Practice and Exam-style questions yourself.

■ The disc method

Revolving a region under $y = f(x)$ around the x -axis makes discs of radius $f(x)$:

$$V = \pi \int_a^b (f(x))^2 dx.$$

■ A worked disc volume

Revolve $y = \sqrt{x}$, $0 \leq x \leq 4$, around the x -axis: radius $\sqrt{x}$, so

$$V = \pi \int_0^4 (\sqrt{x})^2 dx = \pi \int_0^4 x dx = \pi \cdot 8 = 8\pi.$$

■ Around the y-axis

Revolving around the y -axis uses $x = f(y)$ and integrates in y : $V = \pi \int_c^d (x(y))^2 dy$.

For $y = x^2$ (so $x = \sqrt{y}$) from $y = 0$ to 4: $V = \pi \int_0^4 (\sqrt{y})^2 dy = \pi \int_0^4 y dy = 8\pi$.

■ Radius is a distance

The radius is always the **distance from the curve to the axis** of revolution —here just $f(x)$ or $x(y)$ because the axis is a coordinate axis.

2 Practice

Now apply the methods above.

2.1 Write the disc-method integral for revolving $y = x$ on $[0, 2]$ about the x -axis. [1]

2.2 Evaluate $\pi \int_0^2 x^2 dx$. [2]

2.3 Find the volume when $y = \sqrt{x}$ on $[0, 9]$ is revolved about the x -axis. [3]

3 Exam-style questions

3.1 Revolving $y = f(x)$ about the x -axis gives the volume [1]

- **A** $\int_a^b f(x) dx$
- **B** $\pi \int_a^b f(x) dx$
- **C** $\pi \int_a^b (f(x))^2 dx$
- **D** $2\pi \int_a^b f(x) dx$

3.2 The region under $y = x^2$ from $x = 0$ to $x = 2$ is revolved about the x -axis.

(a) Write the volume integral. [2]

(b) Evaluate it.

[3]

3.3 The region bounded by $x = y^2$, $y = 0$ to $y = 2$, and the y -axis is revolved about the y -axis. Find the volume. [4]

4 Go further

You are now ready for the real exam questions on this subtopic:

- work through the **8.9 Volumes by the Disc Method** lesson on the **Learn** page;
- read the **Volume with Disc Method: Revolving Around the x- or y-Axis** section of the AP Calculus BC handout on the **Know** page.

Solutions

2.1 $\pi \int_0^2 x^2 dx.$

2.2 $\pi \left[\frac{x^3}{3} \right]_0^2 = \frac{8\pi}{3}.$

2.3 $\pi \int_0^9 x dx = \pi \left[\frac{x^2}{2} \right]_0^9 = \frac{81\pi}{2}.$

3.1 C —the disc method integrates $\pi(\text{radius})^2$.

3.2 (a) $\pi \int_0^2 (x^2)^2 dx = \pi \int_0^2 x^4 dx.$ (b) $\pi \left[\frac{x^5}{5} \right]_0^2 = \frac{32\pi}{5}.$

3.3 radius = $x = y^2$; $V = \pi \int_0^2 (y^2)^2 dy = \pi \int_0^2 y^4 dy = \pi \left[\frac{y^5}{5} \right]_0^2 = \frac{32\pi}{5}.$