Linear Momentum

AP Physics 1

Linear Momentum

Linear momentum 动量 is mass times velocity –a vector pointing the same way as the velocity:

$$\vec{p} = m\vec{v}.$$

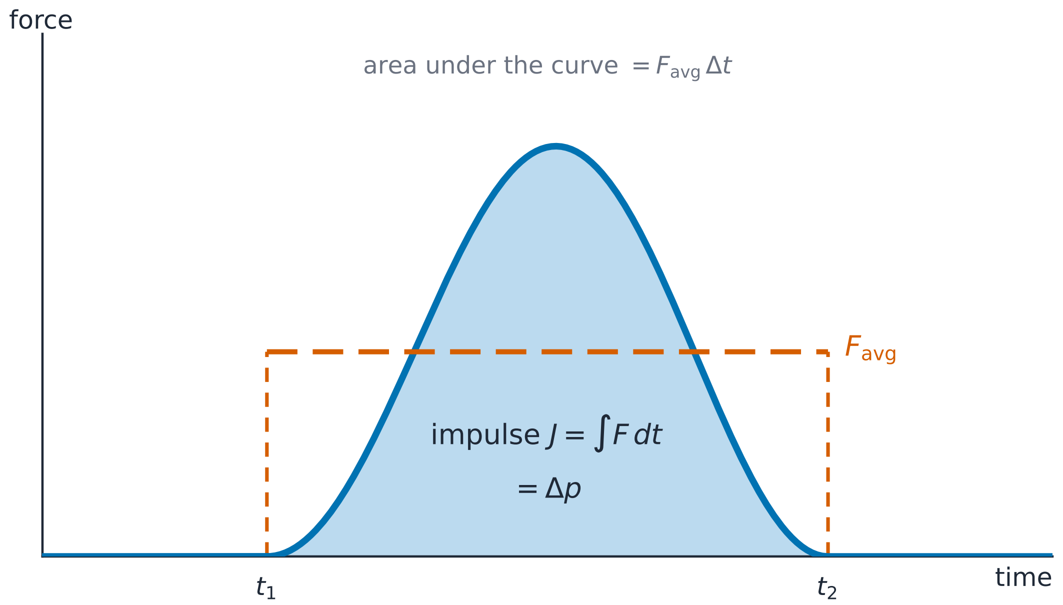
It measures "how hard it is to stop" a moving object. A heavy slow truck and a light fast ball can have the same momentum. Its unit is kg m/s, the same as N s.

Change in Momentum and Impulse

A net force acting over time changes momentum. The **impulse 冲量** delivered is

$$\vec{J} = \vec{F}\,\Delta t = \Delta\vec{p}.$$

This is the **impulse–momentum theorem**: impulse equals the change in momentum. On a force–time graph, impulse is the **area** under the curve. It explains why airbags and follow-through help –spreading the same momentum change over a longer time reduces the force.



Impulse is the area under the force-time curve, equal to the average force times the contact time

Worked example. A 0.15 kg ball hits a wall at 20 m/s and bounces straight back at 15 m/s. The contact lasts 0.020 s. Find the average force on the ball. Take the rebound direction as positive, so $u = -20$ m/s and $v = +15$ m/s:

$$\Delta p = m(v-u) = 0.15\big(15-(-20)\big) = 5.25 \text{ kg m/s}, \qquad F = \frac{\Delta p}{\Delta t} = \frac{5.25}{0.020} = 260 \text{ N}.$$

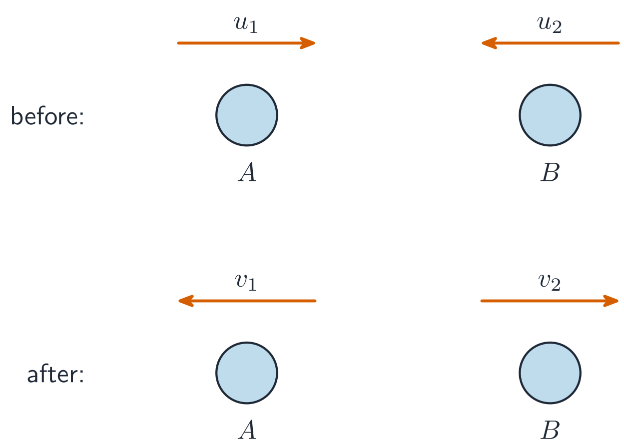
The sign work matters: forgetting that the velocity **reverses** is the most common mistake here.

Conservation of Linear Momentum

If the **net external force** on a system is zero, its total momentum is **conserved** 守恒:

$$\sum \vec{p}_{\text{before}} = \sum \vec{p}_{\text{after}}.$$

Internal forces (like the push between two colliding carts) come in third-law pairs and cancel, so they cannot change the total momentum. This is the key tool for **collisions** 碰撞 and explosions –apply it separately in the x and y directions.



A head-on collision: total momentum before equals total momentum after

Worked example (recoil 反冲). A 60 kg skater, initially at rest on frictionless ice, throws a 2.0 kg ball at 8.0 m/s. Find her recoil speed. The total momentum starts at zero and stays zero:

$$0 = (60)v + (2.0)(8.0) \;\Rightarrow\; v = -\frac{16}{60} = -0.27 \text{ m/s},$$

so she moves at 0.27 m/s in the opposite direction to the ball –the principle behind rockets and guns.

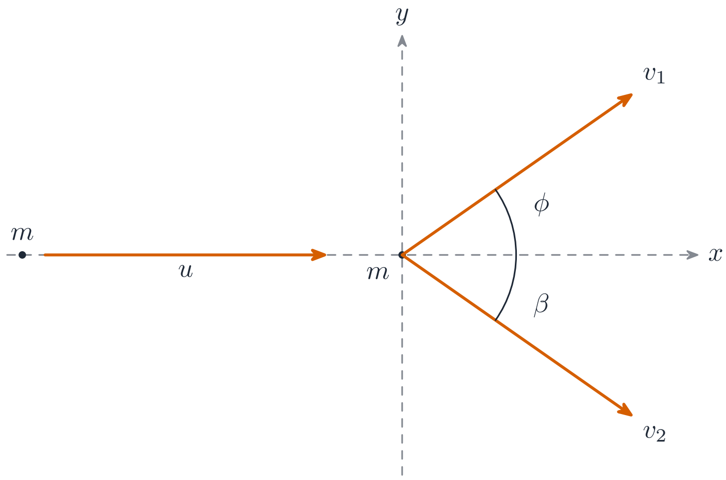
A whole collection of objects can be described by a single **center-of-mass velocity** 质心速度:

$$\vec{v}_{\text{cm}} = \frac{\sum m_i \vec{v}_i}{\sum m_i} = \frac{\vec{p}_{\text{total}}}{M_{\text{total}}}.$$

Since total momentum is conserved when no net external force acts, $\vec{v}_{\text{cm}}$ then stays **constant** –a v_{cm}-versus-time graph is flat unless an outside force acts. An internal collision or explosion never changes it, however violently the parts fly apart.

Elastic and Inelastic Collisions

Momentum is conserved in **every** collision (with no external force). Kinetic energy is not:



A glancing collision, resolved along two perpendicular axes

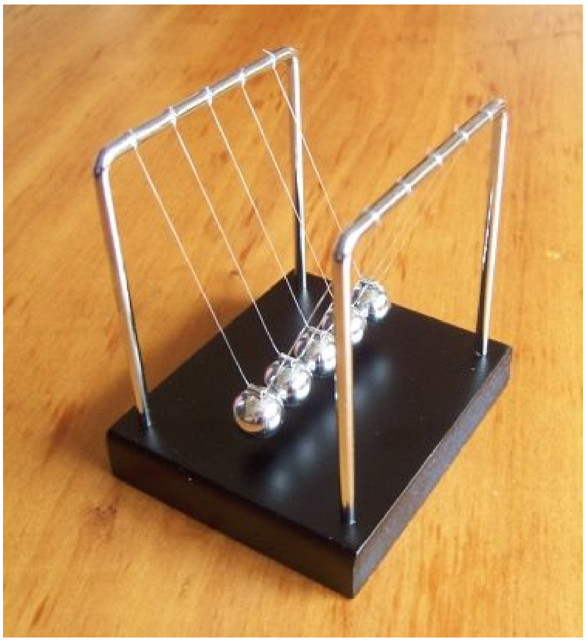
- In an **elastic collision** 弹性碰撞, kinetic energy is **also** conserved (objects bounce apart cleanly).
- In an **inelastic collision** 非弹性碰撞, some kinetic energy becomes heat or deformation. In a **perfectly inelastic** collision the objects **stick together** and move with one common velocity afterward.

Strategy: always write momentum conservation; add energy conservation only if the collision is stated to be elastic. One elastic fact worth memorising: in a 1D elastic collision between **equal masses**, the two objects simply *swap velocities* (a moving ball striking an identical stationary one stops dead, and the target flies off at the incoming speed).

Worked example. A 1000 kg car moving at 20 m/s runs into a stationary 1500 kg car and they lock together. Find their common speed, and the kinetic energy lost. Momentum conservation gives

$$1000 \times 20 = (1000 + 1500)\, v \;\Rightarrow\; v = \frac{20000}{2500} = 8.0 \text{ m/s}.$$

Kinetic energy before is $\frac{1}{2}(1000)(20^2) = 2.0 \times 10^5$ J; after is $\frac{1}{2}(2500)(8.0^2) = 8.0 \times 10^4$ J. So 1.2×10^5 J (about 60%) is lost to crumpling and heat –momentum is still conserved, but kinetic energy is not.



A Newton's cradle shows momentum and kinetic energy passing through a line of balls in a near-elastic collision

Exam tips

- Momentum is a **vector** —assign +/− signs before adding; a ball that **rebounds** reverses its velocity, giving a large Δp.
- Momentum is conserved in **every** collision (no external force); kinetic energy is conserved **only** if the collision is stated to be **elastic**.
- In a **perfectly inelastic** collision the objects stick and move with one common velocity.
- Impulse $= F\,\Delta t = \Delta p =$ the **area** under a force–time graph; spreading a collision over a longer time reduces the force (airbags, bending knees).
- For recoil/explosions, set the total momentum equal before and after (often zero before).