Electric Force, Field, and Potential

AP Physics 2

Electric Charge and Electric Force

Electric charge 电荷 is a **fundamental property** of matter, and it comes in two kinds, positive and negative; like charges repel and opposite charges attract. Charge is **conserved** and **quantized** (a multiple of the elementary charge e). The force between two point charges is **Coulomb's law** 库仑定律:

$$F = \frac{kq_1q_2}{r^2},$$

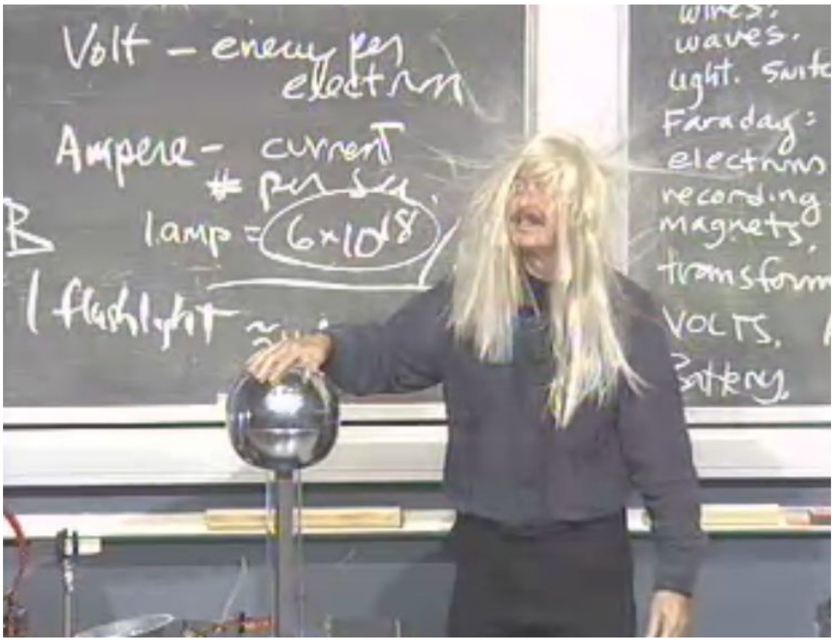
directed along the line joining them –an inverse-square law like gravity, but it can push or pull.

The constant k hides a property of the medium: the **permittivity** 介电常数. Free space has a fixed permittivity of free space ε_0 (with $k = 1/4\pi\varepsilon_0$), while the permittivity of **matter** differs from ε_0, depending on the material's composition and arrangement - which is why a material between two charges changes the force. Note also that although the electric force is vastly **stronger** than gravity, **gravity dominates at large scales**, because large objects are usually electrically **neutral** (equal + and −), leaving only gravity to act.

Worked example. Two point charges, $+3.0\ \mu$C and $-2.0\ \mu$C, sit 0.10 m apart ($k = 9.0 \times 10^9$). The force between them is

$$F = \frac{kq_1q_2}{r^2} = \frac{9.0 \times 10^9 \times (3.0 \times 10^{-6})(2.0 \times 10^{-6})}{(0.10)^2} = 5.4\ \text{N},$$

attractive, because the charges have opposite signs. (Use the magnitudes for the size and decide the direction from the signs.)



Like charges repel: each hair carries the same charge from the Van de Graaff generator, so they push apart

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The Process of Charging

Objects charge by moving electrons. A **conductor** 导体 lets charge move freely; an **insulator** 绝缘体 holds it in place. Three methods:

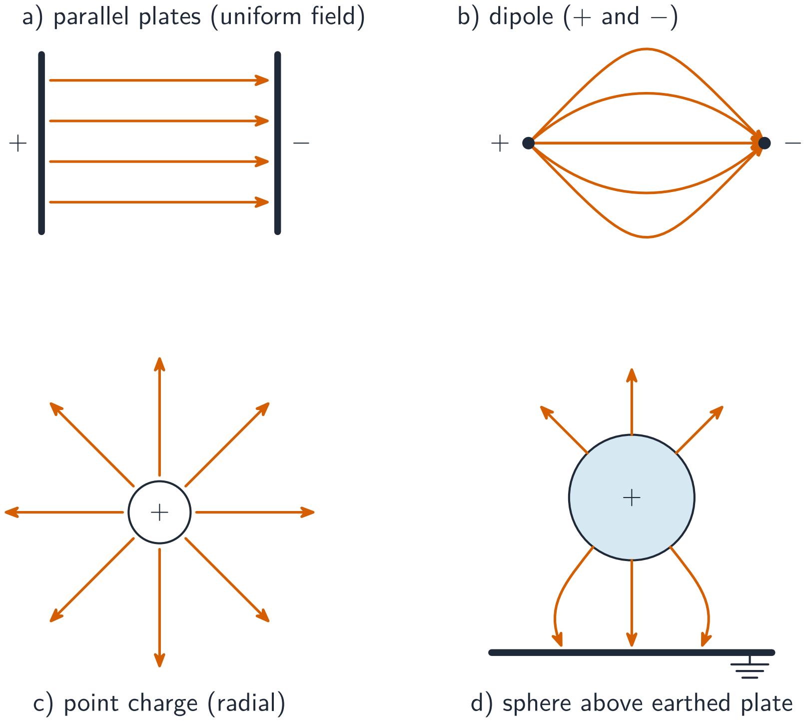
- **Friction**: rubbing transfers electrons.
- **Conduction** 接触起电: touching shares charge.
- **Induction** 感应起电: a nearby charge rearranges charge in a neutral object, which can then be grounded to leave it charged.

Electric Fields

An **electric field** 电场 $\vec{E}$ is the force per unit charge that a small positive test charge would feel:

$$\vec{E} = \frac{\vec{F}}{q}, \qquad E = \frac{kQ}{r^2} \text{ for a point charge.}$$

Field lines point away from positive charges and toward negative ones; where they are denser, the field is stronger. A charge in a field feels $\vec{F} = q\vec{E}$.



Electric field-line patterns for parallel plates, a dipole, and a point charge

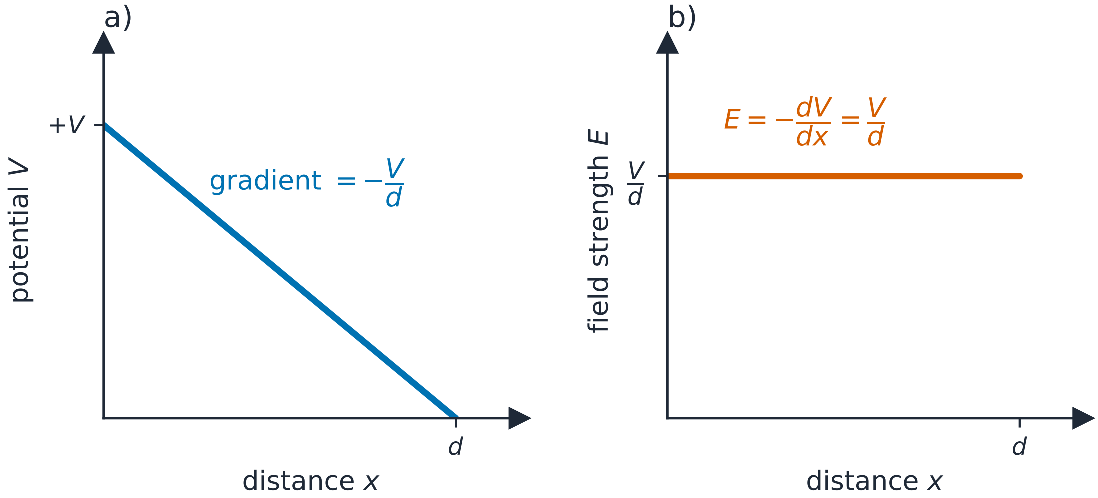
Worked example. Find the electric field 0.20 m from a +5.0 μC point charge: $E = \frac{kQ}{r^2} = \frac{9.0 \times 10^9 \times 5.0 \times 10^{-6}}{(0.20)^2} = 1.1 \times 10^6$ N/C, pointing away from the charge. A +2 nC charge placed there would feel $F = qE = 2 \times 10^{-9} \times 1.1 \times 10^6 = 2.2 \times 10^{-3}$ N.

Electric Potential Energy

Two charges have **electric potential energy 电势能** stored in their arrangement:

$$U = \frac{kq_1q_2}{r}.$$

Like charges pushed together store positive energy; opposite charges have negative energy (bound). Moving a charge changes U, and the electric force does work equal to $-\Delta U$.



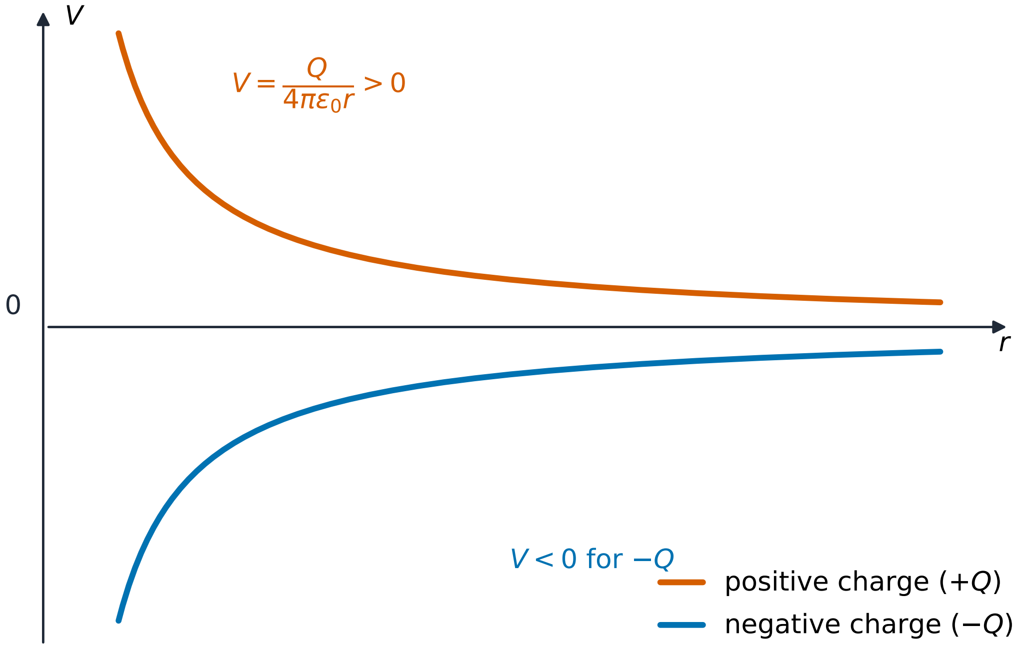
In a uniform field the potential falls steadily with distance, so E relates to V

Electric Potential

Electric potential 电势 V is the potential energy per unit charge –a scalar field, measured in volts:

$$V = \frac{U}{q} = \frac{kQ}{r}.$$

The **potential difference** (voltage) 电压 between two points is the work per unit charge to move between them: $\Delta V = \dfrac{\Delta U}{q}$, and $U = qV$. Positive charges move from high to low potential on their own. Because potential is a **scalar**, adding the potentials from several charges is much easier than adding field vectors.



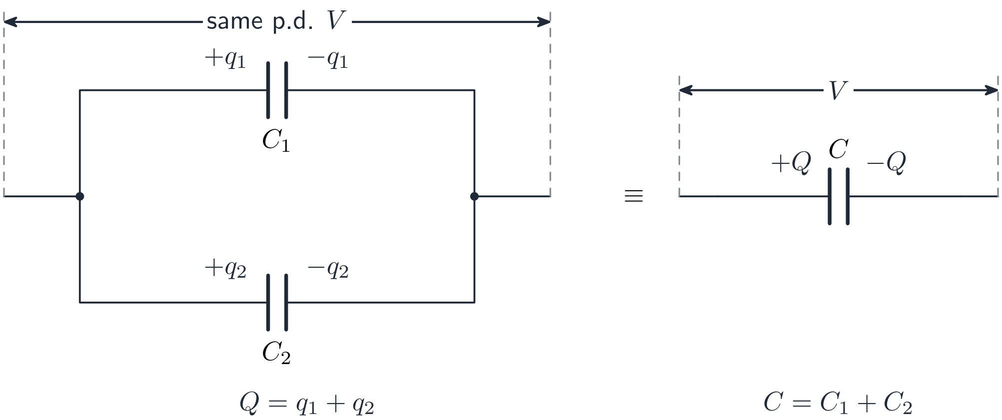
The potential near a point charge varies as 1/r

Capacitors

A **capacitor** 电容器 stores charge and energy on two conductors separated by a gap. Its **capacitance** 电容 relates charge to voltage:

$$C = \frac{Q}{V},$$

and the stored energy is $U = \frac{1}{2}CV^2$. Capacitance depends on the plates' geometry and the material between them, not on the charge placed on it.



Capacitors in parallel share the same p.d., and their charges add

Worked example. A 100 μF capacitor is charged to 12 V. It holds $Q = CV =$

$100 \times 10^{-6} \times 12 = 1.2 \times 10^{-3}$ C of charge and stores $U = \frac{1}{2}CV^2 = \frac{1}{2} \times 100 \times 10^{-6} \times 12^2 = 7.2 \times 10^{-3}$ J of energy.

Conservation of Electric Energy

Energy is conserved for charges just as for masses. A charge released in a field converts electric potential energy into kinetic energy:

$$q\,\Delta V = \tfrac{1}{2}mv^2 \quad \text{(gaining speed as it "falls" through a potential difference).}$$

Worked example. An electron ($q = 1.6 \times 10^{-19}$ C, $m = 9.1 \times 10^{-31}$ kg) is accelerated from rest through a potential difference of 100 V. Its final speed is

$$v = \sqrt{\frac{2q\,\Delta V}{m}} = \sqrt{\frac{2 \times 1.6 \times 10^{-19} \times 100}{9.1 \times 10^{-31}}} = 5.9 \times 10^6 \text{ m/s}.$$

This is exactly how the electron gun in an old television or an electron microscope works.

Exam tips

- **Coulomb's law** and the point-charge field are **inverse-square** —doubling the separation makes the force (or field) four times smaller.
- Use magnitudes for the size of a force and decide direction from the **signs**; the field points the way a **positive** test charge would move.
- **Potential (V) is a scalar** so potentials from several charges simply add; the **field (E) is a vector** and must be added by direction.
- For a charge accelerated through a voltage use energy conservation $qV = \frac{1}{2}mv^2$.
- Capacitor relations: $C = Q/V$ (capacitance is fixed by geometry) and energy $U = \frac{1}{2}CV^2$.