

2.8 Inverse Functions

Name: _____ Class: _____ Date: _____

Total: 9 marks

Objective

Build the skills to answer exam questions on **inverse functions**.

You must be able to:

- find an **inverse function** 反函数 by swapping x and y and solving
- use that the graph of f^{-1} reflects f across the line $y = x$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

■ Inverses

f^{-1} **undoes** f : $(f \circ f^{-1})(x) = x$. Find it by swapping x and y and solving for y . The graph reflects across $y = x$. Only **one-to-one** functions have an inverse function.

■ Example

$f(x) = 2x + 1$: swap $\rightarrow x = 2y + 1 \rightarrow y = \frac{x-1}{2}$, so $f^{-1}(x) = \frac{x-1}{2}$.

2 Practice

2.1 State what an inverse function does. [1]

2.2 State the line of reflection between f and f^{-1} . [1]

2.3 Find the inverse of $f(x) = x + 5$. [2]

3 Exam-style questions

3.1 The graph of f^{-1} is the reflection of f across [1]

- **A** the x -axis
 - **B** the y -axis
 - **C** the line $y = x$
 - **D** the origin
-

3.2 A function has an inverse function only if it is [1]

- **A** increasing
 - **B** one-to-one
 - **C** linear
 - **D** positive
-

3.3 $f(x) = 3x - 6$.

(a) Swap x and y . [1]

(b) Solve for y . [1]

(c) State $f^{-1}(x)$. [1]

4 Go further

- work through the **2.8 Inverse Functions** lesson on the **Learn** page;
- read the **Exponential and Logarithmic Functions** section of the AP Precalculus handout on the **Know** page.

Solutions

2.1 it undoes f , returning the original input.

2.2 the line $y = x$.

2.3 swap: $x = y + 5$; solve: $y = x - 5$, so $f^{-1}(x) = x - 5$.

3.1 C.

3.2 B.

3.3 (a) $x = 3y - 6$. (b) $y = \frac{x+6}{3}$. (c) $f^{-1}(x) = \frac{x+6}{3}$.