4.12 Linear Transformations and Matrices

Name: ____________ Class: ________ Date: __________ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **linear transformations and matrices**.

You must be able to:

- apply a matrix to a vector by **matrix-vector multiplication** 矩阵向量乘法
- describe the geometric effect (scaling, identity)

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Linear transformations

A 2×2 matrix acts on a vector:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}.$$

This transforms the plane —scaling, rotation, or reflection.

Example

$\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 \\ 3 \end{bmatrix} = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$ —a scaling by 2.

2 Practice

2.1 State how a matrix acts on a vector. [1]

2.2 Compute $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 5 \\ 2 \end{bmatrix}$. [1]

2.3 Compute $\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. [2]

3 Exam-style questions

3.1 The identity matrix $\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$ maps a vector to [1]

- **A** the zero vector
- **B** itself
- **C** its reverse
- **D** twice itself

3.2 $\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ applied to a vector gives a [1]

- **A** rotation
- **B** scaling by 3
- **C** reflection
- **D** shift

3.3 $A = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}$.

(a) Compute $A \begin{bmatrix} 1 \\ 4 \end{bmatrix}$. [1]

(b) Compute $A \begin{bmatrix} 3 \\ 0 \end{bmatrix}$. [1]

(c) Describe the effect. [1]

4 Go further

- work through the **4.12 Linear Transformations and Matrices** lesson on the **Learn** page;
- read the **Functions Involving Parameters, Vectors, and Matrices** section of the AP Precalculus handout on the **Know** page.

Solutions

2.1 by matrix-vector multiplication, giving $\begin{bmatrix} ax+by \\ cx+dy \end{bmatrix}$.

2.2 $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

2.3 $\begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$. (b) $\begin{bmatrix} 6 \\ 0 \end{bmatrix}$. (c) it stretches the x-component by 2 and leaves y unchanged.