4.13 Matrices as Functions

Name: ______________ Class: __________ Date: ____________ **Total: 9 marks**

Objective

Build the skills to answer exam questions on **matrices as functions**.

You must be able to:

- view a matrix transformation $T(\vec{v}) = A\vec{v}$ as a **function** 函数 on vectors
- relate composition of transformations to the matrix product

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Matrices as functions

A matrix transformation $T(\vec{v}) = A\vec{v}$ is a function taking an input vector to an output vector. Applying two transformations in sequence corresponds to **multiplying** their matrices: applying A then B is $B(A\vec{v}) = (BA)\vec{v}$.

Example

$A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ swaps components: $T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right) = \begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

2 Practice

2.1 State what the input and output of a matrix transformation are. [1]

2.2 State what composing two matrix transformations corresponds to. [1]

2.3 If $T(\vec{v}) = A\vec{v}$ and $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, find $T\left(\begin{bmatrix} 2 \\ 5 \end{bmatrix}\right)$. [2]

3 Exam-style questions

3.1 A matrix transformation $T(\vec{v}) = A\vec{v}$ takes [1]

- **A** a number to a number
- **B** a vector to a vector
- **C** a matrix to a scalar
- **D** nothing

3.2 Applying transformation A then B equals the single matrix [1]

- **A** $A + B$
- **B** BA
- **C** $A - B$
- **D** AB only if diagonal

3.3 $A = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$.

(a) Find $A \begin{bmatrix} 2 \\ 5 \end{bmatrix}$. [1]

(b) Find $A \begin{bmatrix} 7 \\ 0 \end{bmatrix}$. [1]

(c) Describe the effect. [1]

4 Go further

- work through the **4.13 Matrices as Functions** lesson on the **Learn** page;
- read the **Functions Involving Parameters, Vectors, and Matrices** section of the AP Precalculus handout on the **Know** page.

Solutions

2.1 the input is a vector and the output is a vector.

2.2 multiplying their matrices.

2.3 $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$.

3.1 **B**.

3.2 **B**.

3.3 (a) $\begin{bmatrix} 5 \\ 2 \end{bmatrix}$. (b) $\begin{bmatrix} 0 \\ 7 \end{bmatrix}$. (c) it swaps the two components (a reflection across $y = x$).