6.10 Setting Up a Test for the Difference of Two Population Proportions

Name: ____________ Class: ________ Date: __________ **Total: 8 marks**

Objective

Build the skills to answer exam questions on **setting up a test for a difference of two proportions**.

You must be able to:

- state the hypotheses $H_0 : p_1 = p_2$ and H_a
- explain why a **pooled (combined) proportion** 合并比例 is used
- write the pooled proportion $\hat{p}_c = \dfrac{x_1 + x_2}{n_1 + n_2}$

1 Worked examples

Study these first. Each one shows the method for a question type used later.

Hypotheses

- **Null** $H_0 : p_1 = p_2$ (no difference).
- **Alternative** $H_a : p_1 \neq p_2$ (or one-sided).

The pooled proportion

Because H_0 assumes the proportions are **equal**, we combine both samples to estimate that common value:

$$\hat{p}_c = \frac{x_1 + x_2}{n_1 + n_2},$$

and use $\hat{p}_c$ in the standard error of the test statistic.

2 Practice

2.1 State the null hypothesis for a two-proportion test. [1]

2.2 State why a pooled proportion is used in the test. [1]

2.3 Write the formula for the pooled proportion. [1]

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3 Exam-style questions

3.1 The null hypothesis in a two-proportion test is [1]

- **A** $p_1 \neq p_2$
- **B** $p_1 = p_2$
- **C** $p_1 = 0$
- **D** $\hat{p}_1 = \hat{p}_2$

3.2 The pooled proportion combines [1]

- **A** only group 1
- **B** both groups' successes and sample sizes
- **C** the two means
- **D** nothing

3.3 Group 1 has 30 successes in 100; group 2 has 20 in 100.

(a) Find $\hat{p}_1$. [1]

(b) Find $\hat{p}_2$. [1]

(c) Find the pooled proportion $\hat{p}_c$. [1]

4 Go further

- work through the **6.10 Setting Up a Test for the Difference of Two Population Proportions** lesson on the **Learn** page;
- read the **Inference for Categorical Data: Proportions** section of the AP Statistics handout on the **Know** page.

Solutions

2.1 $H_0 : p_1 = p_2$.

2.2 H_0 assumes the proportions are equal, so both samples estimate the same common value.

2.3 $\hat{p}_c = \dfrac{x_1 + x_2}{n_1 + n_2}$.

3.1 **B**.

3.2 **B**.

3.3 (a) 0.30. (b) 0.20. (c) $\hat{p}_c = \dfrac{30 + 20}{100 + 100} = 0.25$.