# Geometry

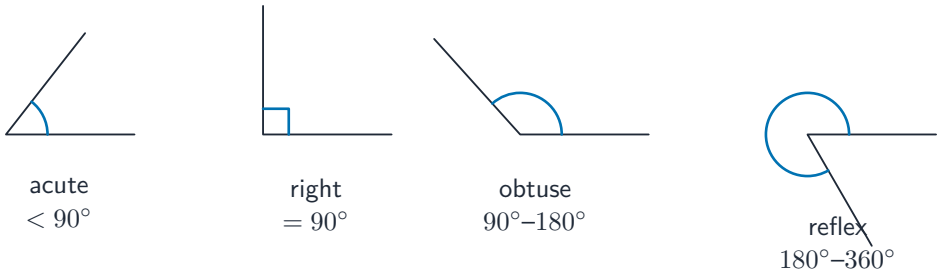
IGCSE Mathematics

This handout covers Topic 4, Geometry. Parts marked **(Extended)** are only tested on the Extended papers; everything else is for both levels. In the exam you must **give reasons** using the correct names below, not just the numbers.

## Lines and angles

A corner where two lines meet is a **vertex** 顶点. Two lines are **parallel** 平行 if they never meet, and **perpendicular** 垂直 if they meet at a right angle. Angles are named by their size:

| Name | Size |
|---|---|
| **acute angle** 锐角 | less than 90° |
| **right angle** 直角 | exactly 90° |
| **obtuse angle** 钝角 | between 90° and 180° |
| **reflex angle** 优角 | between 180° and 360° |




*Angles by size: acute (under* 90°*), right (*90°*, shown by a square), obtuse (*90°–180°*) and reflex (*180°–360°*).*

We name an angle with three letters, e.g. angle $ABC$ is the angle at $B$.

## Angle facts

Learn these basic facts. Each one is a valid reason in the exam.

- Angles at a point add up to 360°.
- Angles on a straight line add up to 180°.
- **Vertically opposite angles** 对顶角 (made by two crossing lines) are equal.

**Worked example.** Three angles on a straight line are $x$, 50° and 70°. Find $x$.

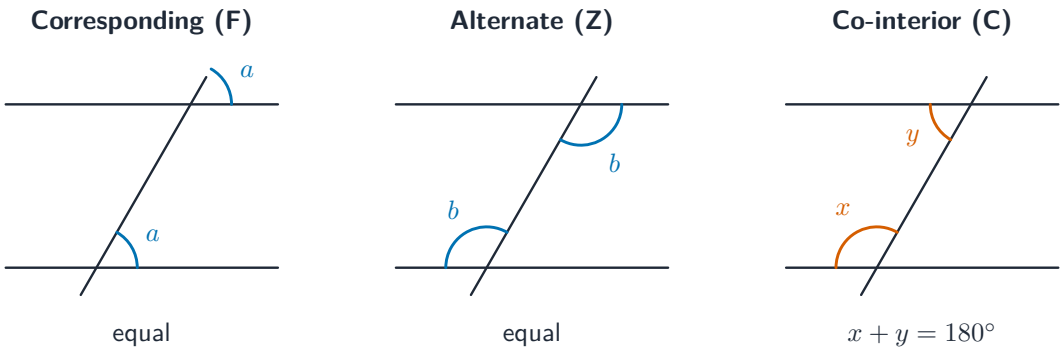
$$x + 50 + 70 = 180 \;\Rightarrow\; x = 60°.$$

# Angles in parallel lines

When a line crosses two parallel lines:

- **Corresponding angles** 同位角 (in matching positions, an "F" shape) are equal.
- **Alternate angles** 内错角 (opposite sides of the crossing line, a "Z" shape) are equal.
- **Co-interior angles** 同旁内角 (a "C" shape) add up to 180°; we say they are **supplementary** 互补.

**Worked example.** A straight line crosses two parallel lines. One angle is 110°. The co-interior angle $y$ satisfies $110 + y = 180$, so $y = 70°$.




*Corresponding (F) and alternate (Z) angles are equal; co-interior (C) angles add to* 180°.

# Triangles

A **triangle** 三角形 has three sides and angles that add up to 180°.

| Type | Property |
|---|---|
| **equilateral** 等边 | all sides equal, all angles 60° |
| **isosceles** 等腰 | two sides equal, two angles equal |
| **scalene** 不等边 | all sides and angles different |
| right-angled | has one right angle |

**Worked example.** A triangle has angles $x$, $2x$ and 90°. Find $x$.

$$x + 2x + 90 = 180 \;\Rightarrow\; 3x = 90 \;\Rightarrow\; x = 30°.$$

# Quadrilaterals

A **quadrilateral** 四边形 has four sides and angles that add up to 360°.

| Shape | Property |
|---|---|
| **square** 正方形 | four equal sides, four right angles |
| **rectangle** 矩形 | opposite sides equal, four right angles |
| **parallelogram** 平行四边形 | opposite sides parallel and equal |
| **rhombus** 菱形 | four equal sides, opposite sides parallel |
| **kite** 鸢形 | two pairs of equal sides next to each other |
| **trapezium** 梯形 | one pair of parallel sides |

# Polygons

*A honeycomb tessellates the plane with regular hexagons.*



A **polygon** 多边形 is a shape with straight sides. A **regular polygon** 正多边形 has all sides and all angles equal.

| Sides | Name |
|---|---|
| 5 | **pentagon** 五边形 |
| 6 | **hexagon** 六边形 |
| 8 | **octagon** 八边形 |
| 10 | **decagon** 十边形 |

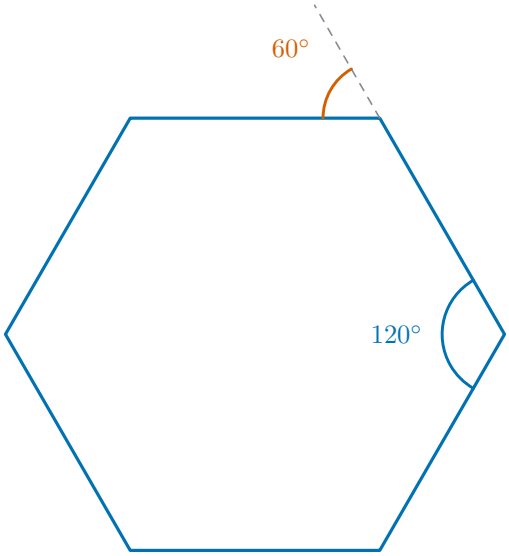
For a polygon with $n$ sides:

$$\text{sum of interior angles} = (n-2) \times 180^\circ, \qquad \text{sum of exterior angles} = 360^\circ.$$

The **interior angle** 内角 and the **exterior angle** 外角 at each corner add up to $180^\circ$.

**Worked example.** Find each interior angle of a regular hexagon.

The exterior angle is $\dfrac{360^\circ}{6} = 60^\circ$, so each interior angle is $180^\circ - 60^\circ = 120^\circ$.




$$\text{interior} + \text{exterior} = 180°$$

*At each corner the interior and exterior angles add to* 180°*; a regular hexagon has* 60° *exterior and* 120° *interior angles.*

## Symmetry

*The Taj Mahal has a clear line of symmetry down its centre.*

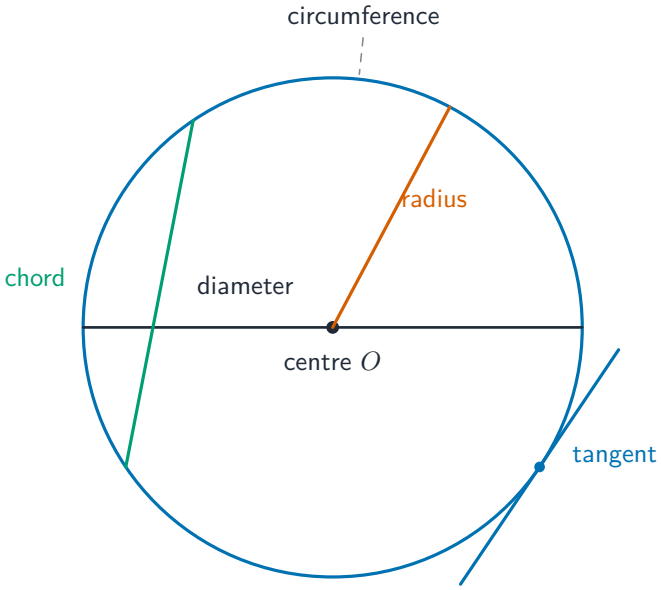
Image: Jonathan Freundlich, CC BY-SA 3.0 (commons.wikimedia.org)

- A shape has **line symmetry** 轴对称 if a mirror line splits it into two matching halves.
- A shape has **rotational symmetry** 旋转对称 if it fits onto itself as you turn it. The order is how many times it fits in one full turn.

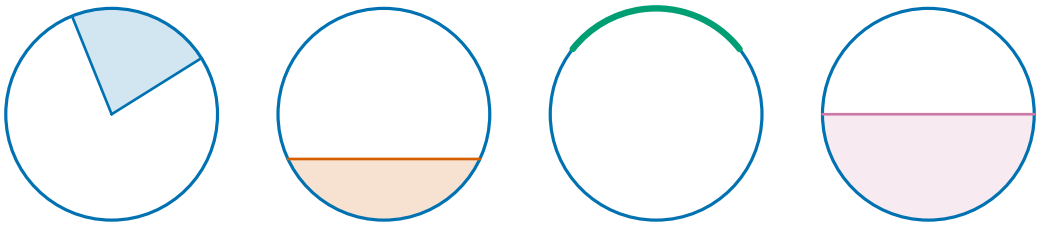
For solids (**Extended**), a flat slice that splits the solid into mirror halves is a **plane of symmetry** 对称面, and a line you can spin it around is an **axis of symmetry** 对称轴.

# Circles: the parts

| Term | Meaning |
|---|---|
| **circle** 圆 | all points the same distance from a centre |
| **centre** 圆心 | the middle point |
| **radius** 半径 | centre to edge (plural *radii*) |
| **diameter** 直径 | right across through the centre (= 2× radius) |
| **circumference** 圆周 | the distance all the way round |
| **chord** 弦 | a straight line joining two points on the circle |
| **arc** 弧 | part of the circumference |
| **sector** 扇形 | a "pizza slice" between two radii |
| **segment** 弓形 | the region cut off by a chord |
| **semicircle** 半圆 | half a circle |
| **tangent** 切线 | a line that touches the circle at one point |




*The main parts of a circle: a chord joins two points, while a tangent touches at just one.*



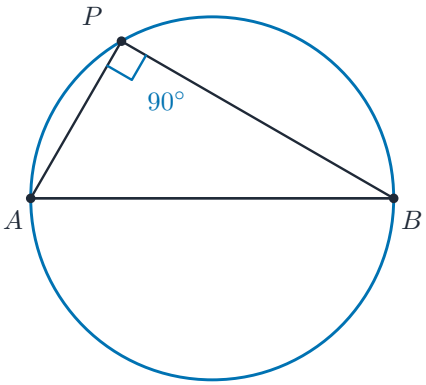

*A sector is a slice between two radii, a segment is cut off by a chord, an arc is part of the circumference, and a semicircle is half the circle.*

# Circle theorems

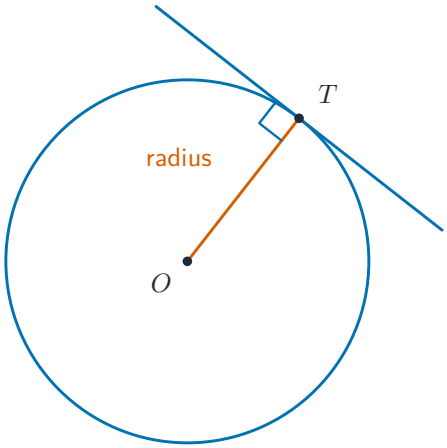
Use these to find unknown angles, always giving the reason.

**For both levels:**

- The angle in a **semicircle** is 90°.
- The angle between a **tangent** and a **radius** is 90°.




**angle in a semicircle** = 90°




**tangent meets radius** = 90°

*Two theorems for both levels: the angle in a semicircle is* 90°*, and a tangent meets a radius at* 90°*.*

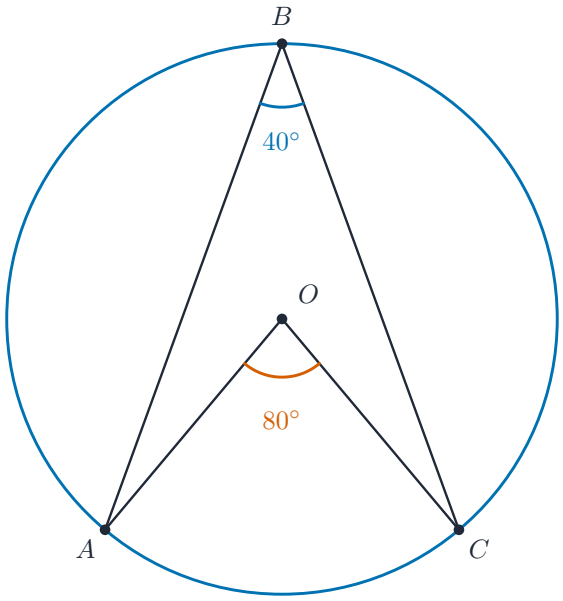
**Extended (theorems I):**

- The angle at the **centre** is twice the angle at the **circumference** (standing on the same arc).
- Angles in the same **segment** are equal.
- Opposite angles of a **cyclic quadrilateral** 圆内接四边形 add up to 180°.
- The **alternate segment theorem** 弦切角定理: the angle between a tangent and a chord equals the angle in the other segment.

**Extended (theorems II):** equal chords are the same distance from the centre; the perpendicular bisector of a chord passes through the centre; two tangents from the same outside point are equal in length.

**Worked example.** A, B, C are on a circle. The angle at the circumference $ABC$ is 40°. Find the angle $AOC$ at the centre $O$.

The angle at the centre is twice the angle at the circumference: $2 \times 40° = 80°$.




angle at centre $= 2\times$ angle at circumference

*The angle at the centre ($80°$) is twice the angle at the circumference ($40°$) standing on the same arc.*

## Similar shapes

Two shapes are **similar** 相似 if one is an enlargement of the other: same angles, and all sides multiplied by the same **scale factor** 比例因子 $k$. (Shapes that are exactly the same size and shape are **congruent** 全等.)

For similar shapes and solids:

$$\frac{\text{area of } A}{\text{area of } B} = k^2, \qquad \frac{\text{volume of } A}{\text{volume of } B} = k^3.$$

**Worked example.** Two similar solids have lengths in the ratio $2 : 3$. The smaller has volume $40\ \text{cm}^3$. Find the volume of the larger.

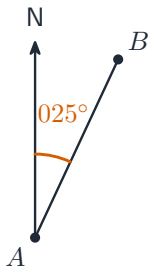
The volume ratio is $2^3 : 3^3 = 8 : 27$. So the larger volume is $40 \times \dfrac{27}{8} = 135\ \text{cm}^3$.

## Bearings

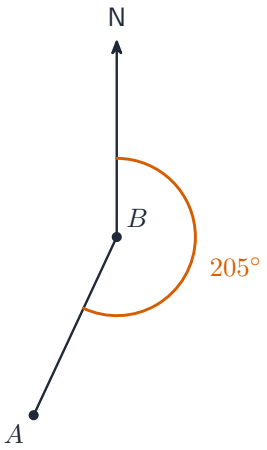
A **bearing** 方位角 gives a direction as a three-figure angle, measured **clockwise** 顺时针 from north, from $000°$ to $360°$. So due east is $090°$ and due south is $180°$.

**Worked example.** The bearing of $B$ from $A$ is $025°$. Find the bearing of $A$ from $B$.

The return (back) bearing differs by $180°$: $025° + 180° = 205°$.




**Bearing of $B$ from $A$**




**Bearing of $A$ from $B$**

*A bearing is measured clockwise from north as three figures; the back bearing differs by* 180°.

## Constructions, nets and solids

- To **construct** 作图 a triangle from three given sides, draw the base with a ruler, then use a pair of **compasses** 圆规 to mark each other side as an arc. Leave the construction arcs showing.
- A **net** 展开图 is a flat shape that folds up into a solid. You can use a net to work out surface areas.
- A **scale drawing** 比例图 shows a real object smaller (or larger) by a fixed scale, such as 1 cm to 5 m.

Common **solids** 几何体 and their parts:

| Solid | Note |
|---|---|
| **cube** 立方体 / **cuboid** 长方体 | box shapes |
| **prism** 棱柱 | the same shape all along its length |
| **cylinder** 圆柱 | a circular prism |
| **pyramid** 棱锥 / **cone** 圆锥 | come to a point |
| **sphere** 球 / **hemisphere** 半球 | a ball / half a ball |
| **frustum** 平截头体 | a cone or pyramid with the top cut off |

A flat side of a solid is a **face** 面, two faces meet at an **edge** 棱, and the whole outside is its **surface** 表面.