Mensuration

IGCSE Mathematics

This handout covers Topic 5, Mensuration (measuring length, area and volume). The Core and Extended content here is almost the same. In the exam, some formulas are given in the List of formulas, but you should still learn them all.

Units of measure

We use **metric** 公制 units for **mass** 质量 (g, kg), **length** 长度 (mm, cm, m, km), **area** 面积, **volume** 体积 and **capacity** 容量 (ml, litres —the space inside a container).

To change units, be careful with squares and cubes:

- Length: 1 m = 100 cm.
- Area: 1 $\text{m}^2 = 100^2 = 10\,000$ cm^2.
- Volume: 1 $\text{m}^3 = 100^3 = 1\,000\,000$ cm^3.
- Capacity: 1 litre = 1000 cm^3, so 1 m^3 = 1000 litres.

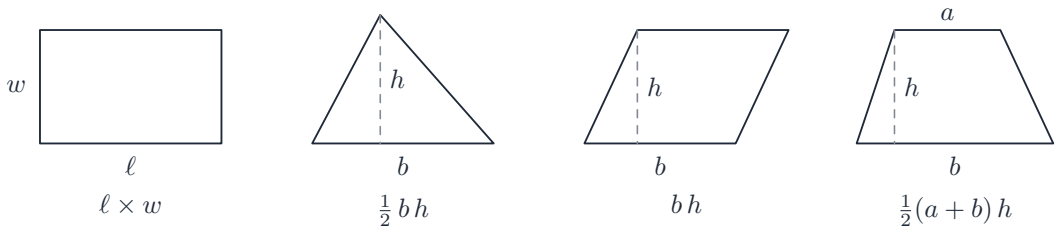
Worked example. Convert 3 m^2 to cm^2.

$$3 \times 10\,000 = 30\,000 \text{ cm}^2.$$

Perimeter and area of basic shapes

The **perimeter** 周长 is the distance all the way round a shape. The **area** is the amount of flat space inside it. Here b is the **base** 底 and h is the perpendicular **height** 高.

Shape	**Area**
rectangle 矩形	length × width
triangle 三角形	$\frac{1}{2} \times b \times h$
parallelogram 平行四边形	$b \times h$
trapezium 梯形	$\frac{1}{2}(a+b)h$, where a and b are the two parallel sides



Area of the basic shapes; b is the base, h the perpendicular height, and a, b the two parallel sides of a trapezium.

Worked example. A trapezium has parallel sides 6 cm and 10 cm, and height 4 cm. Find its area.

$$\tfrac{1}{2}(6+10)\times 4 = \tfrac{1}{2}\times 16\times 4 = 32 \text{ cm}^2.$$

Circles

For a **circle** 圆 with **radius** 半径 r (and **diameter** 直径 $d = 2r$):

$$\text{circumference} = 2\pi r = \pi d, \qquad \text{area} = \pi r^2.$$

The **circumference** 圆周 is the distance round the circle.

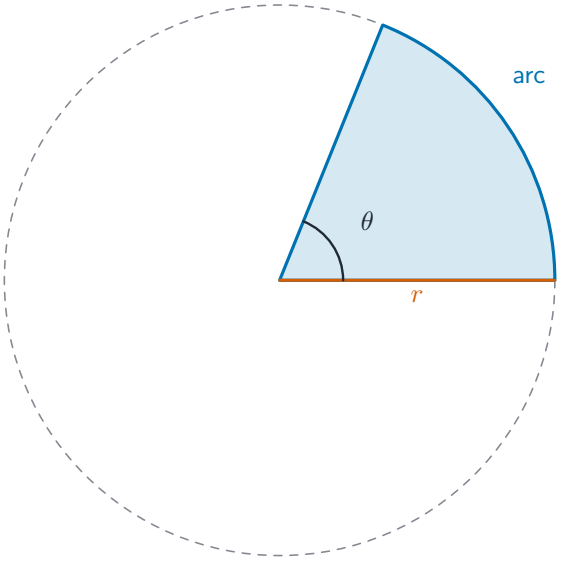
Worked example. A circle has radius 7 cm. Find its circumference and area (leave π in the answer).

$$\text{circumference} = 2\pi \times 7 = 14\pi \text{ cm}, \qquad \text{area} = \pi \times 7^2 = 49\pi \text{ cm}^2.$$

Arcs and sectors

An **arc** 弧 is part of the circumference. A **sector** 扇形 is a "pizza slice" between two radii. If the sector angle is θ, the arc and sector are that fraction $\dfrac{\theta}{360}$ of the whole circle:

$$\text{arc length} = \frac{\theta}{360}\times 2\pi r, \qquad \text{sector area} = \frac{\theta}{360}\times \pi r^2.$$



A sector is the fraction $\frac{\theta}{360}$ of the whole circle, so its arc and area are that fraction of the circumference and the area.

A small slice is a **minor sector** 小扇形; the large rest is a **major sector** 大扇形.

Worked example. Find the **arc length** 弧长 and area of a sector with angle 90° and radius 8 cm.

The fraction is $\dfrac{90}{360} = \dfrac{1}{4}$, so

$$\text{arc} = \tfrac{1}{4} \times 2\pi \times 8 = 4\pi \text{ cm}, \qquad \text{area} = \tfrac{1}{4} \times \pi \times 8^2 = 16\pi \text{ cm}^2.$$

Surface area and volume of solids

The pyramids of Giza are square-based pyramids —a 3-D solid.

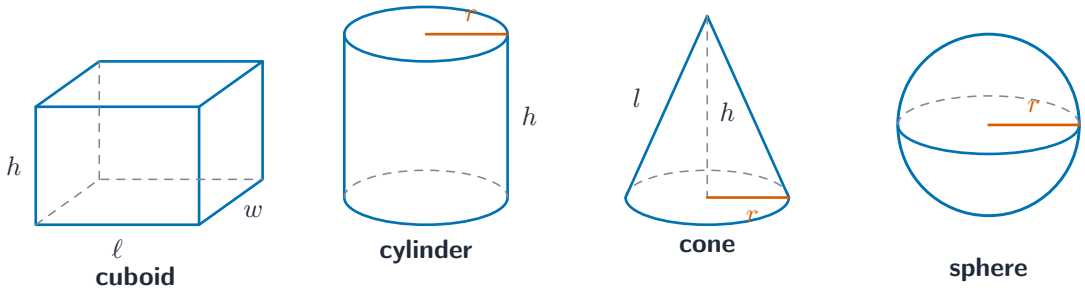
Image: Ricardo Liberato, CC BY-SA 2.0 (commons.wikimedia.org)

The **surface area** 表面积 is the total area of all the outside faces. The volume is the space inside. For these solids (r = radius, h = height):

The pyramids of Giza are giant square-based pyramids —real solids whose volume and surface area you can calculate

Image: Ricardo Liberato, CC BY-SA 2.0 (commons.wikimedia.org)

Solid	Volume	Surface area
cuboid 长方体	length × width × height	add the six faces
prism 棱柱	(**cross-section** 横截面 area) × length	—
cylinder 圆柱	$\pi r^2 h$	$2\pi rh$ (**curved surface area** 侧面积) $+ 2\pi r^2$
pyramid 棱锥	$\frac{1}{3}\times$ base area $\times h$	—
cone 圆锥	$\frac{1}{3}\pi r^2 h$	πrl (curved) $+ \pi r^2$, where l is the **slant height** 斜高
sphere 球	$\frac{4}{3}\pi r^3$	$4\pi r^2$



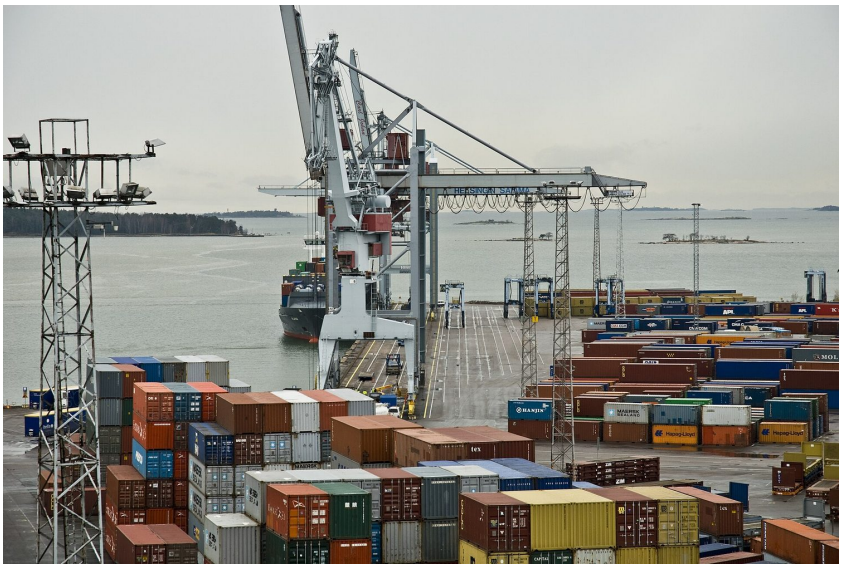
Common solids and the dimensions (r, h, ℓ, slant l) used in their volume and surface-area formulas.

Worked example. A cylinder has radius 5 cm and height 10 cm. Find its volume and total surface area (in terms of π).

$$\text{volume} = \pi \times 5^2 \times 10 = 250\pi \text{ cm}^3.$$

$$\text{surface area} = 2\pi(5)(10) + 2\pi(5)^2 = 100\pi + 50\pi = 150\pi \text{ cm}^2.$$

Compound shapes and parts of shapes



Stacked containers are cuboids; volume is length times width times height.

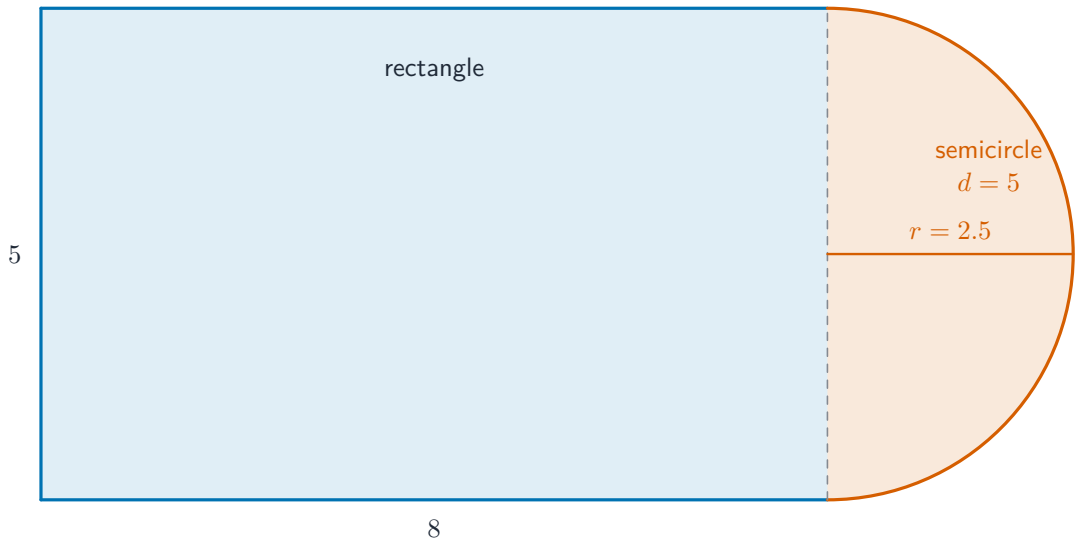
A **compound shape** 组合图形 is made by joining or cutting simple shapes. Split it into parts you know, then add or subtract.

For "parts" of a circle or solid, take the right fraction. For example, a **hemisphere** 半球 (half a sphere) has volume

$$\tfrac{1}{2} \times \tfrac{4}{3}\pi r^3 = \tfrac{2}{3}\pi r^3.$$

A **frustum** 平截头体 is a cone or pyramid with its top cut off; find its volume by subtracting the small top cone from the whole cone.

Worked example. Find the area of a shape made of a rectangle 8 cm × 5 cm with a semicircle of diameter 5 cm on one end.



Split a compound shape into parts you know —here a rectangle plus a semicircle —then add the areas.

The semicircle has radius 2.5 cm:

$$\text{area} = 8 \times 5 + \tfrac{1}{2}\pi(2.5)^2 = 40 + 3.125\pi \approx 49.8 \text{ cm}^2.$$