Probability

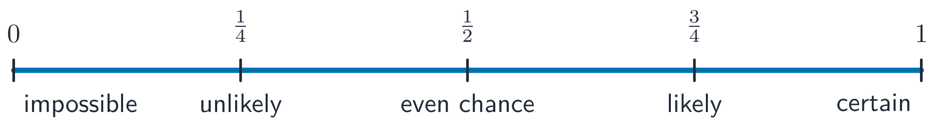
IGCSE Mathematics

This handout covers Topic 8, Probability. Parts marked **(Extended)** are only tested on the Extended papers; everything else is for both levels.

The probability scale

Dice: the probability scale runs from impossible (0) to certain (1).

Probability 概率 measures how likely an **event** 事件 is. It runs on a scale from 0 (impossible) to 1 (certain), and can be written as a fraction, decimal or percentage. We write $\mathrm{P}(A)$ for the probability of event A.



Probability runs from 0 *(impossible) to* 1 *(certain), with* $\frac{1}{2}$ *an even chance.*

For equally likely **outcomes** 结果,

$$\mathrm{P}(\text{event}) = \frac{\text{number of favourable outcomes}}{\text{total number of outcomes}}.$$

The probability that an event does **not** happen is

$$\mathrm{P}(A') = 1 - \mathrm{P}(A).$$

Worked example. A bag has 3 red and 5 blue counters. Find the probability of *not* drawing red.

$$P(\text{red}) = \frac{3}{8}, \qquad P(\text{not red}) = 1 - \frac{3}{8} = \frac{5}{8}.$$

Relative frequency and expected frequency

When outcomes are not equally likely, do an experiment. The **relative frequency** 相对频率 estimates the probability:

$$\text{relative frequency} = \frac{\text{number of times it happened}}{\text{total number of trials}}.$$

The more trials you do, the better the estimate. A **fair** 公平 object gives equal chances; one with **bias** 偏倚 does not; **random** 随机 means each outcome happens by chance.

The **expected frequency** 期望频数 is how many times you expect an event in n trials:

$$\text{expected frequency} = P(\text{event}) \times n.$$

Worked example. The probability of rolling a six is $\frac{1}{6}$. How many sixes are expected in 300 rolls?

$$\frac{1}{6} \times 300 = 50.$$

Combined events

A roulette wheel: each number is an equally likely outcome.

For two or more events together (**combined events** 组合事件), two rules help:

- **AND** (both happen): **multiply** the probabilities —when the events are **independent** 独立事件 (one does not affect the other).

- **OR** (either happens): **add** the probabilities —when the events are **mutually exclusive** 互斥 (they cannot both happen).

Three pictures help you organise the work.

Sample space diagrams

A **sample space diagram** 样本空间图 is a table or grid listing every possible outcome —useful for two dice or two spinners. Count the outcomes you want out of the total.

die 2

die 1 / +	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

the six highlighted cells all give a total of 7

A sample space diagram lists every outcome; for the total of two dice there are 36 *equally likely cells.*

Venn diagrams

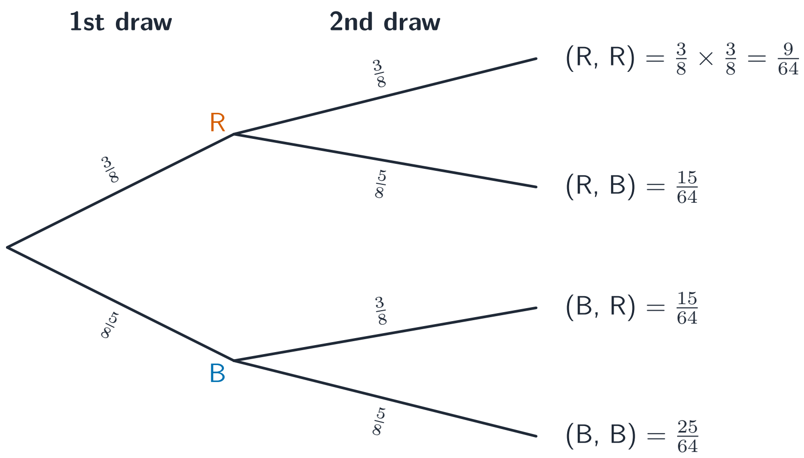
A **Venn diagram** 维恩图 sorts outcomes into overlapping sets. From it you can read $\mathrm{P}(A \cap B)$ (in both) and $\mathrm{P}(A \cup B)$ (in either).

Tree diagrams

A **tree diagram** 树状图 shows each stage as a set of branches. Write the probability on each branch and the outcome at the end. **Multiply along** the branches, then **add** the paths you want.

Worked example (with replacement). From the bag (3 red, 5 blue), a counter is drawn, replaced, then a second is drawn. This is **with replacement** 有放回, so the chances do not change. Find the probability of two reds.

$$\mathrm{P}(\text{red, red}) = \frac{3}{8} \times \frac{3}{8} = \frac{9}{64}.$$



On a tree diagram, multiply the probabilities along the branches; the four outcomes' probabilities add to 1*.*

Worked example (without replacement, Extended). Now the first counter is *not* replaced —drawing **without replacement** 无放回. After one red is taken, 2 reds remain out of 7:

$$\mathrm{P(red,\ red)} = \frac{3}{8} \times \frac{2}{7} = \frac{6}{56} = \frac{3}{28}.$$

Conditional probability (Extended)

Conditional probability 条件概率 is the probability of one event *given that* another has already happened. Read it from a Venn diagram, two-way table or tree diagram by looking only at the part that matches the condition.

Worked example. In a class of 30, 18 study French and, of those, 7 also study German. A French student is picked. Find the probability they also study German.

Look only at the 18 French students:

$$\mathrm{P(German} \mid \mathrm{French)} = \frac{7}{18}.$$